Inflation Targeting under Fiscal Fragility

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Abstract

We study the inflation target level decision under a high government debt burden. Lower targets allow for lower on-target inflation but increase the temptation to use extra inflation to generate fiscal revenue. We model this trade-off in an economy with an altruistic policymaker choosing debt and public expenditure on behalf of private agents whom finance expenditures and form inflation expectations rationally. For low debt levels, the inflation target is credible and always delivered. For high debt, the target is not credible and never delivered. In-between, the target may be delivered or not, leading to above-target expectations and expensive debt rollovers. We show that rollover costs are lower when the inflation target is higher. Our model implies that the optimal inflation target depends on debt levels.

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1 Introduction

The inflation targeting regime became the cornerstone of central bank coordination of inflation expectations after its widespread adoption in the 1980s and 1990s. However, inflation target ranges are missed frequently in both advanced and emerging economies. Episodes of coordination failures in which inflation expectations suddenly lose their anchor and diverge from the announced targets are common[1]. Most of these episodes lack sizable changes in fundamentals that would explain the shift in expectations, raising questions on the limits of inflation targeting to anchor short-term inflation expectations. Since the seminal contribution of Calvo (1988), several models that were proposed exhibit multiple equilibria for inflation in a monetary setting and explain how a sudden deterioration in inflation expectations can disanchor inflation from a lower level[2]. However, none of these models analyzed the explicit role of the inflation target level choice in avoiding a self-fulfilling inflationary crisis.

We propose a model that rationalizes these observable episodes of coordination failures and self-fulfilling inflation under an inflation targeting framework in indebted economies. In our model, the inflation target level is central in determining the ability of the policymaker to anchor expectations and avoid loss of confidence on the monetary regime. This allows us to build a theory for setting the inflation target. The heart of our argument is that the economy's fiscal side is fundamental to understanding the capacity of the inflation-targeting regime to coordinate inflation expectations. In our model, a lower inflation target reduces inflation welfare costs, but increases the temptation of the policymaker to deviate to a higher level of inflation that alleviates the fiscal constraints, making the economy more prone to a credibility loss. Higher inflation targets, on the other hand, raise inflation costs, but also raise the confidence in the ability of the policymaker to deliver the target and retain credibility.

We model a closed economy in which two types of agents, an altruistic policymaker and private agents, act rationally in an environment with complete information. The policymaker acts jointly as a fiscal authority and central bank, pursuing an inflation target by choosing current inflation and financing government expenditures by selling debt. We assume that the policymaker is not perfectly committed to the inflation target and might deviate from it to make fiscal room for spending. Its decision is the solution to the tradeoff between inflating public debt away and keeping inflation on target to avoid the economic costs of deviating. Private agents choose how much debt to hold and form expectations about next-period infla-

[1] Roger and Stone (2005) note that targets are often missed (40% in their sample) and sometimes "by substantial amounts and for prolonged periods." Based on our updated data set, used in Appendix B, we conclude that targets are still frequently missed (26%).

[2] See Araujo and Leon (2002), Araujo, Leon, and Santos (2013), Corsetti and Dedola (2016) and Bianchi and Mondragon (2021) for examples of models in which a self-fulfilling crisis can occur in a monetary setting with sovereign debt economies.

tion. Our framework builds on Calvo (1988), Araujo, Leon, and Santos (2013), and mainly Cole and Kehoe (1996, 2000), expanding their analysis to a inflation targeting setting.

In the proposed model, target failures can happen when the public debt level exceeds an endogenous threshold limit and enters the fiscal fragility zone (FFZ). When debt is low enough, the interest burden is low, and therefore government spending is high. When debt is above the endogenous cutoff and within the FFZ, the expected inflation rate is higher than the inflation target, generating a higher cost of debt service and lower government spending. If the benefit of abandoning the inflation target exceeds the cost of maintaining it, the policymaker will inflate the public debt and increase public spending. Within the FFZ, the policymaker is subject to confidence crises, so the optimal policy may be to gradually reduce the public debt, eliminating confidence crises and supporting the inflation-targeting regime. The inflation target plays a critical role: higher targets can avoid a confidence crisis and facilitate the exit of the FFZ, mitigating the risk of missing the target. Inflation-indexed bonds can also play a role in increasing confidence in the target, albeit with a welfare cost by making crises more severe should they happen.

The intuition for why a higher inflation target helps to anchor expectations is simple. We show that higher targets reduce the marginal benefit of a higher inflation level, as the policymaker faces a higher inflation expectation. As a result, the amount of partial default available, the ratio between the inflation chosen by the policymaker when deviating and the inflation target, decreases at the target level. This in turn makes maintaining the target less costly, and deviating from the target less beneficial, in the FFZ. Consequently, the endogenous lower limit that characterizes the FFZ increases with the inflation target level. We conclude that the optimal inflation target is the lowest possible such that the economy exits the FFZ. The intuition for adopting indexed bonds is similar. They raise credibility by increasing the cost of deviating from the target[3].

Our paper provides practical implications for the design of the monetary policy framework. It seems naïve to choose a 2% inflation target without considering fiscal fundamentals as countries eventually do. Our model suggests that a higher inflation target helps to coordinate expectations in a scenario with high government debt burden. Our model enlightens the fact that missing the target is more frequent in indebted economies, and also why they tend to have higher targets. It also may bring some light in the recent episode in which developed economies have missed the inflation target after they were led to increase debt in response to the recent pandemic.

[3] In the Appendix, we test our model predictions using a panel dataset of 20 countries with at least 15 years of inflation targeting. We find evidence that the size of deviations from the target and the probability of deviating are negatively related to the target level. We also find evidence that deviations from the target are positively related to the debt level.

We calibrate our model based on the response to inflationary pressures in Brazil at the end of 2002. In that period, it became clear that the presidential candidate who would win the election could arrive with a new policy framework. As a result, inflation expectations exceeded the upper bound of the target, as seen in Figure I, indicating a target confidence crisis. In response to rising inflation expectations, Brazilian policymakers twice increased the target for 2003, first at an additional meeting held in June 2002 and again in January 2003. This response by policymakers is in line with the recommendations that can be inferred from our model.

Insert Figure I about here.

Related Literature: Our article contributes to the literature on the ability of the inflation targeting regime to anchor inflation expectations and provide a low and stable inflation level for indebted economies. It is the conventional view, synthesized on the seminal work of Woodford (2003), that monetary policy can stabilize inflation around a predetermined target path, so long as the monetary authority commits to a specific policy trajectory. Under this view, the debate over the inflation target centers around the welfare maximizing level of inflation, considering different specifications for nominal rigidities and taxation distortions of the economy, such as in Schmitt-Grohé and Uribe (2010) and Adam and Weber (2019). In this framework, the optimal inflation target ranges from a negative level associated with the Friedman rule, up to a positive level consistent with models with a zero-lower bound in nominal interest-rates and sticky prices. While this framework is adequate to study the problem of setting a target in economies in a credible fiscal path, the same may not be true for countries with a high debt burden in which inflation serves as a partial default mechanism. We add a formulation in which the inflation target itself affects monetary policy credibility, so that a higher target may increase welfare by raising the credibility of the inflation targeting regime.

Our article also adds to the message on the fiscal limits of monetary policy and the interdependence between fiscal discipline and price stability, which is amply addressed in the literature. Sargent and Wallace (1981) show the importance of fiscal side to understanding inflation control, followed by Leeper (1991), Sims (1994, 2011), Woodford (1995), Leeper and Leith (2016), Araujo, Berriel, and Santos (2016), Cochrane (2018) and Bianchi, Faccini, and Melosi (2023).

We contribute by incorporating a policymaker that decides monetary policy strategically, who does not follow a Taylor rule and may use inflation to partially default, into a simple DSGE. This approach closely follows papers on confidence crises in debt markets such as Cole and Kehoe (1996, 2000), Calvo (1988), and Arellano, Mihalache, and Bai (2019). While other papers explore debt crises and their relation to monetary policy, such as Uribe (2006), Aguiar, Amador, Farhi, and Gopinath (2013), Corsetti and Dedola (2016), Bacchetta, Perazzi, and van Wincoop (2018), and Arellano, Mihalache, and Bai (2019), none of them consider the relation between fiscal fundamentals and the inflation target level.

Next Sections: In Section 2, we set out the model and derive the recursive form defining the equilibrium. We define the discretionary inflation chosen by the policymaker when deviating from the target and prove that it is increasing in the debt level. We then obtain endogenous debt zones that characterize the behavior of the policymaker, and prove that the optimal policy is stationary outside of the FFZ in which multiple equilibria are possible. We finish the section by proving that the real interest rate is decreasing in the inflation target when the economy is in the FFZ, and that the lower bound of the FFZ is increasing in the target level. In Section 3, we specify functional forms and parameter values in a quantitative analysis to match the situation in Brazil in 2002. We then analyze the results from our model, and obtain the optimal inflation target for each debt level. In the numerical exercises, the optimal target is the lowest possible level in which the economy exits the FFZ. In Section 4, we analyze the 2002 confidence crisis in Brazil and the subsequent policy responses, and show that the actions taken by the government to anchor inflation expectations are consistent with the results predicted by our model. Finally, the last section presents concluding remarks.

2 Model

We consider a closed economy with two types of agents: a policymaker and private agents. Each agent lives infinite periods and forms rational expectations with complete information. The policymaker acts as a combined fiscal and monetary authority, choosing current inflation and selling one-period debt to finance itself. In our setup, the inflation choice reduces to a discrete choice each period of whether to deviate from a pre-announced inflation target. We assume that the policymaker is altruistic and maximizes private agent welfare. Private agents receive a stream of fixed endowments. In each period, they choose how much debt to hold and form expectations about next-period inflation while considering the inflation target and the current debt level. When multiple equilibria are possible, a sunspot variable determines the equilibrium.

2.1 Basic Setup

Policymaker

We consider an altruistic policymaker who chooses both fiscal and monetary policies to maximize private agents' utility. The policymaker, as a monetary authority, chooses the inflation rate π_t, which generates disutility, and, as a fiscal authority, the next period's debt D_{t+1}:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t, g_t) - \psi(\pi_t) \right] \tag{1}$$

where c_t is private agents' consumption in period t, g_t is government spending on public goods, and β is the intertemporal discount rate, $0 < \beta < 1$. Consumption and public goods are nonnegative. We define private agents' utility as a weighted average of linear consumption and government spending utility similar to Cole and Kehoe (2000). The weights are defined by the parameter $\rho \in (0, 1)$ that can be interpreted as a relative preference for consumption:

$$u(c_t, g_t) = \rho c_t + (1 - \rho) v(g_t)$$

where v is a twice-differentiable strictly increasing and strictly concave function of g satisfying $\lim_{g \to 0^+} v(g) = -\infty$.

Linearity in consumption is a strong assumption and deserves further comment. As will be explained further, the policymaker makes decisions strategically considering the impact of its actions on the equilibrium real interest rate. The linearity assumption implies that the real rate is independent of private consumption levels, which, otherwise, would be an additional variable for the policymaker to track in its inflation and debt decision. In other words, it results in an equilibrium real interest rate that depends only on the inflation level and the inflation target. This simplification makes the problem significantly more tractable both analytically and computationally. Linearity in consumption also simplifies the problem by making the debt stationary outside the crisis zone (to be defined) and readily makes the marginal utility of public goods higher than the marginal utility of consumption since public spending is constrained by debt interest spending and the fixed tax rate.

The disutility of inflation ψ is assumed to satisfy $\psi' > 0$ and $\psi'' \geq 0$. It captures the welfare and output costs of higher inflation levels[4] and is independent of the remaining variables of the model, except, possibly, of the inflation target π^a that will be defined later. Convexity of the disutility function is a necessary assumption to guarantee an interior solution for the

[4] Under a linear consumption utility specification, a utility cost of inflation can be interpreted as a cost in terms of consumption goods. See Bailey (1956) and Lucas (2000) for examples of models that find a way for inflation levels to affect welfare and output. Cysne (2009) shows that Bailey's measure provides a measure of the welfare costs of inflation derived from an intertemporal general-equilibrium model, while Campos and Cysne (2018) estimate output costs of inflation for the Brazilian case.

discretionary inflation chosen by the policymaker[5]. When the disutility function depends on the target, we also assume that $\psi'' + \frac{1+\pi^a}{1+\pi}\frac{\partial \psi'}{\partial \pi^a} \geq 0$, which implies that the marginal disutility of inflation is non-decreasing on the inflation level when delivering the target. This hypothesis accommodates a wide array of possible disutility functions that may depend on the inflation level, on the target level or on the deviation from the target, such as $\psi(\pi) = \kappa(\pi - \pi^a)^2$, for a constant parameter κ. In the numerical exercises, we specify a simple quadratic cost $\psi(\pi) = \kappa\pi^2$, for simplicity, without compromising general results.

In each period, the policymaker finances the nonnegative spending g_t and the repayments on previous-period obligations through a fixed tax rate τ[6] on a deterministic endowment e and the issuance of new debt D_{t+1}[7]. We assume that the tax level is not high enough so that total tax revenue exceeds the optimal amount of public spending, which is the level that equates to marginal spending between public and private consumption. Since we are appraising the interplay between fiscal fragility and inflation targeting we consider a binding government budget restriction in which public debt is required to raise public spending. Mathematically, we can write this hypothesis as $(1 - \rho)v'(\tau e) \geq \rho$. The marginal utility of spending in public consumption is larger than the one of spending in private consumption, which is equal to ρ given our linear utility in consumption. The government's budget constraint is given by

$$g_t + (1 + r_t)D_t \leq D_{t+1} + \tau(e - \epsilon_t) \, , \tag{2}$$

where D_t is the last-period debt, and r_t is the real interest rate[8] [9].

The fixed endowment is subject to a penalty ϵ_t that depends on the policymaker's choice of inflation. Let π^a denote the exogenously set inflation target, assumed to be set in the adop-

[5]In the Online Appendix C we also consider a variation of the model in which inflation decreases the endowment level instead of being a utility cost, with similar results.

[6]The fixed tax rate hypothesis can be interpreted as a situation in which the policymaker has no additional space to increase taxes to reduce indebtedness without significantly affecting output. This situation is similar to what is observed in a middle-income economy with relatively high tax levels such as Brazil. Alternatively, this hypothesis can be interpreted as an impossibility of adjusting taxes during a crisis.

[7]Our emphasis in the paper is on the debt rollover cost. As a result, the proper concept of debt (gross or net) may depend on the context of the country analyzed. For example, in the presence of foreign reserves accumulation, the debt rollover cost will depend on the spread of domestic interest rate over foreign reserves return. For emerging economies, such as the Brazilian economy, which serves as our illustrative case, the return on foreign reserves is small compared to the high-interest rates on government debt, so the gross debt level is a better indicator of indebtedness in the economy. This may not be the case in advanced economies with a lower spread. For simplification, we refrain from explicitly modeling the reserves accumulation decision.

[8]We restrict our analysis to initial debt levels that leave the policymaker with a nonempty set of feasible choices, $D_t \in [0, D^{max}]$, where D^{max} is high enough. For very high initial debt levels, the policymaker could have no way of satisfying the positive constraint on c and g. To see this, suppose that debt servicing costs are higher than tax revenues, $\left(\frac{1}{\beta} - 1\right) D_0 > \tau e$, which leaves no space for spending. Then, even if the policymaker were to partially default on debt payments, it would still be unable to meet future positive spending restrictions due to the high future debt servicing costs and the inability to use inflationary surprises again.

[9]We also assume a no-Ponzi condition, so the government cannot run-up infinite debt in the long run: $\lim_t \beta^t D_{t+1} = 0$.

tion of the inflation targeting regime. Then, ϵ_t is a permanent fixed cost that affects the economy if the policymaker chooses to deviate from the target, reflecting the effect of a loss of credibility on the economy[10]. This fixed cost is of the form

$$\epsilon_t = \begin{cases} 0 & \text{if } \pi_t = \pi^a, \epsilon_{t-1} = 0bvbv \\ \epsilon & \text{otherwise} \end{cases}$$

We consider ϵ to be a high but bounded penalty cost, always significantly higher than the inflation cost when inflation is on the target but limited to avoid unreal perfect commitment. Formally, this implies that $\epsilon > \psi(\pi^a)/\rho$, which means that the penalty for deviating is always higher than the inflation disutility of delivering the target, in terms of private consumption. It is worth mentioning that we are considering a sufficiently high penalty cost to avoid results of target deviations coming from a relaxed penalty assumption.

The ex post real interest rate is given by:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_t} \ , \tag{3}$$

where i_t is the nominal risk-free interest rate on bonds that were issued in period $t - 1$.

In each period, the policymaker can satisfy the budget constraint by i) adjusting expenditures, ii) issuing new debt D_{t+1}, and iii) partially defaulting on debt through an inflationary surprise, π_t, and rolling over the remaining debt. Given an nominal interest i_t defined at t, an inflationary surprise reduces the ex post real interest rate and, consequently, the payments the policymaker makes on its debt. Such a partial default offers additional fiscal room for government spending.

Private Agents

We assume a continuum of infinitely lived private agents who choose consumption and savings to maximize their expected utility:

$$\max_{c_t, d_{t+1}} \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \ , \tag{4}$$

Since private agents do not choose the inflation level, the disutility function ψ is omitted

[10] Our approach of assuming an exogenous functional form for the "cost of deviating" is in line with the literature. Exogenous penalty functions are also assumed in self-fulfilling debt crisis models as in Cole and Kehoe (1996, 2000) and in sovereign default models as in Arellano (2008). A positive credibility cost gives an incentive to the policymaker to follow the inflation target. We interpret the penalty function as a reduced and parsimonious form of capturing the negative impacts of inflation deviation on economic activity.

from their maximization problem. Each period, private agents receive a deterministic endowment e, subject to a credibility penalty ϵ_t, and payments on their bond holdings. The endowment is taxed at a constant rate τ by the government. The private agents' budget constraint is given by:

$$c_t + d_{t+1} \leq (1 + r_t)d_t + (1 - \tau)(e - \epsilon_t) \tag{5}$$

where d_{t+1} represents one-period bonds bought in t and d_t is the previous-period bond holdings paying the interest rate $(1 + r_t)$. The first-order condition for debt demand from the private agents problem implies the following equilibrium nominal interest-rate:

$$1 + i_t = \frac{1}{\beta} \frac{1}{\mathbb{E}_{t-1}\left[\frac{1}{1+\pi_t}\right]} \tag{6}$$

Let $q_t = (1 + \pi_t)^{-1}$ be the inverse gross inflation rate. Private agents also form their expectations $q_t^e = \mathbb{E}_{t-1}q_t$. Since the policymaker has a binary choice of either following the previously announced target π^a or deviating to a discretionary inflation level π^D, the inflation expectation will be equal either to one of these levels or a probability weighted sum of them, depending on the sunspot variable to be introduced. As a result, the expectation q_t^e will have a one-for-one correspondence to the expected inflation level $\pi_t^e = \mathbb{E}_{t-1}\pi_t$. The expectations formed will depend on the timing of actions.

Timing

Rational expectations govern the strategic interactions between the policymaker and private agents. As in Calvo (1988) and Cole and Kehoe (1996, 2000), multiple self-fulfilling equilibria may occur where, conditional on the debt level, the best response from the policymaker's perspective depends on the expectations of private agents. If private agents expect a deviation from the target, the best response will be to deviate. If they expect no deviations, the best response will be to keep inflation on target. In this case, we consider an exogenous sunspot variable ζ_t to determine the selection of the equilibrium. The sunspot variable determines which of the possible inflation rates will be the actual inflation rate π_t implemented by the government when there are two equilibrium rates: the inflation target π^a and discretionary inflation π_t^D.

At the beginning of each period, uncertainty is resolved through the realization of the sunspot variable ζ_t. The policymaker, considering the sunspot variable previously drawn, chooses how much debt D_{t+1} to sell and the inflation rate π_t, which will either be the target π^a or the discretionary inflation rate π_t^D. Finally, private agents form their expectations about the next period's inflation, π^e_{t+1}, which determines the expected inverse gross inflation rate q^e_{t+1}, and consequently nominal interest rate i_{t+1}, and decide the level of debt d_{t+1}. In summary, the timing of the model is:

1^{st} The sunspot variable ζ_t is realized.

2^{nd} The policymaker chooses actual inflation π_t, given sunspot ζ_t.

3^{rd} The policymaker chooses the next debt level D_{t+1}.

4^{th} Private agents form next-period inflation expectations π^e_{t+1} and choose the amount of next-period debt d_{t+1} to hold.

Given this timing, private agents may face uncertainty over which equilibrium will be selected next period when forming their inflation expectations. They will form expectations over the probability of each outcome, considering the exogenous distribution of the sunspot variable that determines the actual inflation rate. Inflation expectations will therefore be $\pi^e_t = f\pi^D_t + (1-f)\pi^a$, which imply an inverse gross inflation expectation of $q^e_t = f\frac{1}{1+\pi^D_t} + (1-f)\frac{1}{1+\pi^a}$, where f is the exogenously determined probability of the policymaker deciding to deviate from the inflation target due to an adverse situation. We interpret this negative sunspot as a change in the political landscape or a foreign event that causes a deterioration in the credibility of the policymaker.

Discretionary Inflation

We motivate the existence of deviations from the inflation target by modeling an altruistic policymaker who might choose an inflation level higher than the inflation target as a way of transferring resources for increasing public spending[11]. In each period, the policymaker may choose to deviate from the inflation target π^a, and private agents understand this when forming their expectations π^e_t. We call the inflation rate chosen by the policymaker when deviating from the target *discretionary inflation.* It is the result of a tradeoff between increasing government spending via higher inflation against the cost of facing a higher disutility of inflation. Let π^D_T be the endogenous and optimal level of discretionary inflation chosen at the time T of the deviation.

We assume that once the policymaker deviates from the inflation target, private agents lose confidence in the commitment of the policymaker to the target. Consequently, after

[11]We do not model mechanisms of partial default on local currency domestic debt other than inflation, although governments have opted for alternatives such as reduction of principal or lower coupons (Reinhart and Rogoff, 2008).

the policymaker deviates, the economy enters a steady state because there is no longer any uncertainty to be resolved. The optimal fiscal policy is to maintain constant debt, such as $D_t = D_{T+1}\, \forall\, t > T$, as shown below in Proposition 2. Finally, the inflation disutility function takes the value $\psi(\pi^D)$ when deviating and remains so thereafter, while the credibility penalty is fixed at $\epsilon_t = \epsilon$ forever. The problem the policymaker resolves when defining the level of discretionary inflation can be written as follows:

$$\max_{\pi, D}\, u(c_T, g_T) + \frac{\beta}{1-\beta} u(c, g) - \frac{1}{1-\beta}\psi(\pi)$$

subject to

$$\begin{aligned} g_T &= \tau(e-\epsilon) - (1 + r_T^D)D_T + D \\ g &= \tau(e-\epsilon) - D\left(\frac{1}{\beta} - 1\right) \\ c_T &= e - \epsilon - g_T \\ c &= e - \epsilon - g \\ 1 + r_T^D &= \frac{1}{\beta}\frac{1}{q_T^e}\frac{1}{1+\pi}. \end{aligned} \tag{7}$$

Where $1 + r_T^D$ is the ex-post real interest rate when the policymaker chooses to deviate. The optimal discretionary inflation level π_T^D is the solution to the problem above given an initial debt level D_T[12]. Given rational expectations, in equilibrium, π_T^D is optimal given π_T^e and vice versa[13].

The necessary first-order condition for D plus the hypothesis of concavity of u with respect to g readily implies that the policymaker will set a stationary debt level D such that the stationary public consumption equals public consumption at time T, that is, $g_T = g$. Consequently, we have that the optimal stationary debt is equal to

$$D = \beta(1 + r_T^D)D_T \tag{8}$$

that is, inflationary surprises ($\pi > \pi_T^e$) reduce steady-state debt, decreasing the debt burden and allowing for higher public consumption both in the short and long-run, which implies a

[12]To avoid unnecessary notation, we drop the time subscript T on inflation and the debt level whenever there is no ambiguity.

[13]We numerically solve this problem by writing it as a fixed point. First, we assume an initial $\pi_{T,0}^e = \pi^a$ and then find the optimal $\pi_{T,1}^D$. We update $\pi_{T,1}^e$ using $\pi_{T,1}^D$ according to the inflation expectation formation process of the private agents. If $\pi_{T,1}^e \neq \pi_{T,0}^e$, the problem is iterated to find the new optimal $\pi_{T,2}^D$ given $\pi_{T,1}^e$. We continue this process until $|\pi_{T,i-1}^e - \pi_{T,i}^e| < \eta$, where η is a small number. The existence of a rational expectation of inflation π^e given the optimal discretionary inflation π^D chosen by the policymaker is shown in Online Appendix D.

steady-state level of public spending of

$$g = \tau(e - \epsilon) - (1 - \beta)(1 + r_T^D)D_T. \tag{9}$$

We can rewrite the above equation as

$$(1 + r_T^D)D_T = \frac{1}{1 - \beta}\left(\tau(e - \epsilon) - g\right), \tag{10}$$

which corresponds to the intertemporal budget constraint of the government: the left-hand side is the real value of government debt due in period T, which must be equal to the present value of future government surpluses. Choosing the inflation level π pins down the stationary spending level g that satisfies equation (10).

To gain intuition for the optimal choice of π, consider now the first-order condition of the policymaker's problem for choosing discretionary inflation. The first-order condition, already substituting for the steady-state level of debt, is

$$(\rho - (1 - \rho)v'(g))\frac{\partial(1 + r_T^D)}{\partial \pi}D_T - \frac{1}{1 - \beta}\psi'(\pi) = 0 \tag{11}$$

The first term represents the net marginal benefit of allocating spending to public goods through the inflationary surprise, which is positive since by assumption $(1 - \rho)v'(g) \geq \rho$ for all feasible g, and the real interest rate is decreasing on the inflation level. The second term is the total intertemporal disutility cost of higher inflation on the economy. The discretionary inflation chosen by the policymaker will balance out the marginal benefit of an increase in the public goods consumption that can be purchased through an increase in inflation with the marginal cost of a permanent higher inflation disutility.

Proposition 1 ***(Discretionary Inflation is Increasing in Debt)****: If π_T^D is an interior solution to the problem (7), the discretionary inflation level π_T^D is increasing in the initial debt level D_T:* $\frac{\partial \pi_T^D}{\partial D_T} > 0$.

Proof: see Appendix A.1.

Higher debt levels increase interest spending, reducing available funds for public consumption, and as a result increasing marginal utility of an additional public consumption unit. It also increases the fiscal space opened by deviating, raising the first term in the left-hand side in (11) for any given inflation level, while the second term, the marginal inflation disutility, is unchanged. The resulting discretionary inflation level that satisfies the first-order condition is higher, reflecting a higher marginal benefit of deviating from the target.

2.2 Recursive Equilibrium

We define a recursive equilibrium where the policymaker and private agents sequentially choose their actions. At the beginning of each period, the aggregate state $s = (D, \pi^e, \zeta, \epsilon_{-1})$ is public since the aggregate debt D, the expected inflation for the current period π^e, and the past penalty ϵ_{-1} have all been determined in the previous period, while the realization of the sunspot variable ζ is determined in the start of the period before the agents decide[14]. The policy choices, π and D', the expected inflation for the next period $\pi^{e\prime}$, and the individual debt holdings for the next period d' determine the equilibrium jointly with s. We denote by $\pi(\cdot)$ and $D(\cdot)$ the inflation and debt policy functions, by $r(\cdot)$ the real interest rate function, and by $\pi^e(\cdot)$ the inflation expectation function, all yet to be defined.

To define a recursive equilibrium, we work backward on the timing of actions in each period. We start the definition of a recursive equilibrium with private agents because they move last. When forming expectations $\pi^{e\prime}$ at the end of any period, private agents know all their public debt holding d, the aggregate state s, the policymaker's offer of new debt D', current-period inflation π, and the policymaker's optimal policy functions. The following functional equation defines a private agent's value function:

$$V^{pa}(s, d, \pi, D') = \max_{c,d'} u(c, g) + \beta \mathbb{E} V^{pa}(s', d', \pi', D'')$$

subject to

$$\begin{aligned}
&c + d' \leq (1 + r(s, \pi))d + (1 - \tau)(e - \epsilon(\pi, \pi^a, \epsilon_{-1})) \\
&s' = \Big(D', \pi^e(s, d, \pi, D'), \epsilon(\pi, \pi^a, \epsilon_{-1}), \zeta'\Big) \\
&\pi' = \pi(s') \\
&D'' = D(s') \\
&c \geq 0 \\
&d' \geq 0
\end{aligned} \tag{12}$$

in which we assume that private agents cannot sell public debt. The penalty function $\epsilon(\cdot)$ is a function of its previous value ϵ_{-1}, the inflation target π^a, and current inflation π.

Each period after the policymaker decides how much debt D' to offer and the inflation rate π, private agents decide how much debt to hold. Let $d'(s, d, \pi, D')$ be their debt policy function. When forming inflation expectations, private agents determine the nominal interest rate for the next period. In the absence of multiple equilibria, they perfectly anticipate π, and the real return is always $1/\beta$. If multiple equilibria are possible, private agents do not know

[14] We let $\zeta = 1$ denote the occurrence of the negative sunspot, which is described below, and $\zeta = 0$ otherwise.

what the policymaker will opt to do.

When forming inflation expectations, private agents consider what the policymaker could do in the next period. Their expectations are defined as $\pi^e(s, d, \pi, D') = \mathbb{E}\pi(s')$, where the expectation is conditional on all information available to the agent at the moment. When forming expectations, the set $(D', \pi^{e\prime}, \epsilon) \in s'$ is known to private agents. Hence, the only unknown variable on which private agents form their expectations is the realization of the sunspot variable ζ'. Integrating out the sunspot variable's commonly known distribution, we have

$$\mathbb{E}\pi(s') = \begin{cases} f \times \pi^D(D', \pi^{e\prime}, \epsilon) + (1-f) \times \pi^a & \text{if multiple eq.} \\ \pi^D(D', \pi^{e\prime}, \epsilon) & \text{if deviating unique eq.} \\ \pi^a & \text{if not deviating unique eq.} \end{cases} \tag{13}$$

where f is the exogenous probability of the adverse equilibrium occurring and $\pi^D(D', \pi^{e\prime}, \epsilon)$ is the discretionary inflation chosen by the government when deviating given $(D', \pi^{e\prime}, \epsilon) \in s'$.

The policymaker chooses, at the beginning of the period, inflation π and debt issuance D', given state s. The policymaker knows that the next period's debt level affects the private agents' inflation expectations and resolves the following problem:

$$\begin{aligned} V^p(s) &= \max_{\pi, D'} u\Big(c(s, d, \pi, D'), g\Big) - \psi(\pi) + \beta \mathbb{E} V^p(s') \\ &\text{subject to} \\ &\qquad g + (1 + r(s, \pi))D \leq D' + \tau(e - \epsilon(\pi, \pi^a, \epsilon_{-1})) \\ &\qquad s' = (D', \pi^e(s, d, \pi, D'), \epsilon(\pi, \pi^a, \epsilon_{-1}), \zeta') \\ &\qquad g \geq 0 \end{aligned} \tag{14}$$

We can now define a recursive equilibrium for our model economy. An equilibrium is a list of value functions for the representative private agent V^{pa} and for the policymaker V^p; functions $c(\cdot)$ and $d'(\cdot)$ for the private agents' consumption and saving decisions; functions $\pi(\cdot)$ and $D'(\cdot)$ for the policymaker's inflation and debt decisions; an inflation expectation function $\pi^e(\cdot)$; a real interest rate function $r(\cdot)$; and an equation of motion for the aggregate debt level D' such that the following holds:

- Given D' and π, V^{pa} is the value function for the solution to the representative private agents' problem with c, d' and $\pi^{e\prime}$ being the maximizing choices when $d' = D'$.
- Given q^e, V^p is the value function for the solution to the policymaker problem, and both D' and π are the maximizing choices.

- $D'(s)$ equals $d'(s, d, \pi, D')$.

Our definition of an equilibrium is similar to that of Cole and Kehoe (1996) and Cole and Kehoe (2000) and is restricted to a Markov equilibrium. Future conditional plans of the agents can be derived from their policy functions.

2.3 The Fiscal Fragility Zone

The ability of the policymaker to effectively target inflation is restricted by debt levels. Assuming that inflation has always been on target, three different scenarios can be drawn according to the debt level D[15]:

- The *no crisis zone* $[0, \underline{D}]$: D such that $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D) \rightarrow \pi_t = \pi^a = \pi^e$.

- The *fiscal fragility zone* $[\underline{D}, \overline{D}]$: D such that $\pi \in \{\pi^a, \pi^D\}$ depends on the sunspot.

- The *fiscal dominance zone* $[\overline{D}, D^{max}]$: D such that $V^p(D, \pi_t = \pi^D, \pi^e = \pi^a) \geq V^p(D, \pi_t = \pi^a, \pi^e = \pi^a) \rightarrow \pi_t = \pi^D = \pi^e$.

In the first case, the policymaker finds it preferable to keep inflation on target even when private agents believe that it will not. Consequently, only one equilibrium is possible where private agents have faith in the policymaker delivering on the target inflation. Since there is only one optimal choice for the policymaker regardless of the private agents' expectations, the sunspot ζ is disregarded, and the only important variable defining the policymaker's value function V^p is the debt level D. The same holds for the third case when the only equilibrium is the policymaker always deviating from the inflation target.

Whenever the policymaker is in the no-crisis zone or the fiscal dominance zone, it will always choose a stationary debt policy, as shown in Proposition 2:

Proposition 2 ***(Stationary Policy Outside of the Fiscal Fragility Zone):*** *The optimal debt policy chosen by the policymaker outside of the FFZ at period S is stationary, that is, $D_t = D_S$ for all $t \geq S$.*

Proof: see Appendix A.2.

[15] With slight abuse of notation, we denote by $V^p(D, \pi, \pi^e)$ the total intertemporal utility attained by the policymaker by choosing inflation level π, given debt level D and private agents' expected inflation π^e, assuming that $\epsilon_{-1} = 0$ and that the sunspot $\zeta = 0$.

When the policymaker deviates, private agents lose confidence forever in the ability of the government to maintain inflation targeting and, therefore, expect discretionary inflation, $\pi^e = \pi^D$. It is as if the economy enters the fiscal dominance zone. The above proposition justifies why we are allowed to consider only a stationary debt policy in the definition of discretionary inflation (7), as the fiscal dominance zone is identical to the steady-state once the policymaker deviates from the target. On the other hand, in the no crisis zone, agents expect that the policymaker will follow the target and it is optimal for the government to do so. In this case, due to the strict concavity of the utility function with respect to government spending, it is optimal for policymaker to maintain debt level and spending constant, as agents dislike fluctuating government spending.

The more interesting scenario is multiple equilibria akin to self-fulfilling target failures. If private agents believe that the target will be delivered, then the policymaker will prefer to do so. On the contrary, in the face of adverse expectations, the policymaker chooses to deviate. In this zone, private agents have doubts about the commitment of the monetary authority to the target. The equilibrium is chosen by the realization of a sunspot, something the government binds its choice to but is unrelated to any observable fundamentals, and can be interpreted as a change in the political landscape or a foreign event that causes a deterioration in the credibility of the policymaker and inflation expectations to increase.

When government debt is within the FFZ, the discretionary inflation rate is higher than the inflation target, as shown in Proposition 3:

Proposition 3 ***(Deviations from the Target are Positive):*** *In the Fiscal Fragility Zone the optimal deviation $\pi^D - \pi^a$ is always positive.*

Proof: see Appendix A.3.

If agents believe that the government will deviate to a discretionary inflation level $\pi^D > \pi^a$, then expected inflation π^e is higher than the target, generating a higher cost of debt service and lower government spending to maintain the inflation target. To see this, let us recall the real interest rate on bonds from Equation (3). The real interest rate in the FFZ when the policymaker delivers the target will be given by:

$$1 + r_t^a = \frac{1}{\beta} \frac{1}{f \frac{1+\pi^a}{1+\pi^D} + 1 - f} \tag{15}$$

which is higher than the interest rate outside this zone, $1/\beta$. Thus, the government has an incentive to raise spending in the FFZ, which it can do through inflation or increasing debt.

The credibility cost ϵ plays a critical role in determining the existence of the debt zones and, ultimately, the inflation choice by the policymaker. If it is too low, there is no incentive for the policymaker to deliver the target and the optimal choice will be to deviate do the discretionary inflation π^D regardless of debt level. In this case all debt levels will be in the fiscal dominance zone. On the other hand, if the cost is too high, the welfare of deviating will be too low, as the supply of both private and government consumption will be significantly reduced. As a result, the policymaker will have no incentive to deviate, and agents will anticipate this outcome and set their expectations to the target, so that all debt levels will be in the no-crisis zone. Therefore, a moderate level for the credibility cost will generate a discontinuity on the value function as a function of debt, such that for moderate debt levels there will be multiple equilibria: the fiscal fragility zone.

The Inflation Target Coordination Role

A policymaker facing a higher inflation target has a lower ability to raise spending through deviating from the target, while it faces a lower cost of delivering the target, in the FFZ, as shown in Proposition 4:

Proposition 4 ***(Debt Rollover Cost is Decreasing in the Target Level when Delivering the Target):*** *Let $1 + r_t^a$ denote the real interest rate in the FFZ when the policymaker delivers the inflation target, and $1 + r_t^D$ when it deviates to the discretionary inflation. Then, $\frac{\partial(1+r_t^a)}{\partial\pi^a} < 0$ and $\frac{\partial(1+r_t^D)}{\partial\pi^a} > 0$.*

Proof: see Appendix A.4.

The intuition for this result is as follows: conditional on a moderate credibility cost ϵ, a higher target induces the policymaker to raise π_T^D to attain the same level of reduction in interest expenditure for a given level of debt. Private agents anticipate the higher target and discretionary inflation and demand a higher (nominal) interest rate to purchase government bonds. Recall that the optimal stationary debt choice when deviating is given by

$$D = \frac{1}{f + (1-f)\frac{1+\pi^D}{1+\pi^a}} D_T.$$

Given a higher target π^a, the reduction in real debt provided by an additional inflation rate is diminished, since it is proportional to $1/(1+\pi^a)$. This implies a lower marginal benefit of raising inflation. But since $\psi'' \geq 0$, the marginal cost of an additional inflation rate is nondecreasing. A decreasing marginal benefit combined with a nondecreasing marginal cost results in a lower discretionary inflation. As a result, expected inflation is closer to the target, so that real rates raise when the policymaker deviates and fall when it delivers the target.

As a consequence, a higher inflation target can coordinate better inflation expectations toward the target rate. Faced with a lower benefit of deviating from the target, the policymaker will be less susceptible to inflate. Private agents rationalize this outcome, and set their inflation expectations to the target, which makes delivering the target more accessible for the policymaker. The economy may exit the FFZ if the initial debt level is not too high. This is summarized in the following Proposition:

Proposition 5 ***(Fiscal Fragility Zone Floor raises with Higher Target):*** *The lower limit of the FFZ $\underline{D}$ is increasing in the inflation target level π^a.*

Proof: see Appendix A.5.

Propositions 4 and 5 depend crucially on the fact that

$$\frac{\partial \frac{1+\pi^D}{1+\pi^a}}{\partial \pi^a} < 0.$$

By taking the natural logarithm on the ratio $1+\pi^D/1+\pi^a$ and using the fact that $\log(1+x) \approx x$ for small x, we see that, up to an approximation, this result is equivalent to a decreasing deviation $\pi^D - \pi^a$ as the target is raised.

2.4 Inflation-Indexed Debt

It is not unusual for governments to issue inflation-indexed bonds. We will examine the implications of changing the nature of the bonds. To achieve such indexed bonds within the framework of our model, we change the action timing to give private agents all the needed information to perfectly anticipate policymaker decisions. By allowing private agents to know the realization of the sunspot variable when forming their inflation expectations, bonds will pay a real interest rate $1/\beta$ in all states of nature.

1^{st} The policymaker chooses actual inflation π_t.

2^{nd} The policymaker chooses next debt level D_{t+1}.

3^{rd} The next-period sunspot variable ζ_{t+1} is realized.

4^{th} Private agents form next-period inflation expectations π^e_{t+1} and choose the amount of next-period debt d_{t+1} to hold.

With this new timing, private agents' information sets are given by $(s, d, \pi, D', \zeta') = s'$. Inflation expectations π^e given information set s' will be such that $\pi^e(s') = \pi(s')$ is the policymaker's choice of inflation for the next period[16]. As the policymaker's choices are anticipated, it is no longer possible to transfer resources from private agents in the event of a bad sunspot. In equilibrium, the policymaker would choose $\pi^D = \pi^e$ in the discretionary equilibrium since π^D is optimal given π^e and vice versa. Note that, in equilibrium, discretionary inflation could be different from the announced target $\pi^D \neq \pi^a$. The only way that discretionary inflation could equal the announced target $\pi^D = \pi^a$ would be if the inflation expectation equals the inflation target $\pi^e = \pi^a$. The next section will exploit the differences between indexed and nominal bonds.

3 Quantitative Analysis

In this section, we calibrate the model based on the 2002 confidence crisis in Brazil. The presidential election of 2002 is an interesting case study in that the candidate most likely to win was running on a platform that appeared likely to deteriorate the fiscal situation. Professional forecasters surveyed by the central bank predicted inflation exceeding the target for all horizons. This loss of the credibility of the inflation target in the face of a perceived fiscally fragile situation is the type of event our model is designed to capture.

3.1 Functional Forms and Calibration

In the calibrated model, we assume that the government spending utility function is $v(g) = \log(g)$, and that inflation disutility function is given by $\psi(\pi) = \kappa\pi^2$, where κ is a constant. We consider a disutility function that is independent of the target in order to obtain an ex-ante optimal inflation target for each initial debt level.

Our model is calibrated on a yearly frequency to match the usual time frame targeted by central banks, and almost all parameters correspond to observable values during the 2002 confidence crisis in Brazil. First, we set the inflation target, π^a, to 3.5%, the official target that prevailed in 2002. Second, the discount factor, β, is $1/1.0928$ to match the historical average of the ex post real interest rate between 1996 and 2019[17]. Third, the tax rate on endowments, τ, equals the 2002 general government revenue over GDP, 0.35. Fourth, the exogenous crisis probability, f, matches the country risk captured by the EMBI + Brazil around October 2002.

[16] The different timing only changes problem (7) with respect to how we update inflation expectations π^e.

[17] Using inflation-indexed bonds, such as Brazilian Bonds NTN-C or NTN-B, around 2002 would yield similar results.

Fifth, we set the endowments, e, to 1.5 so that the public spending marginal utility of the total tax revenues is not lower than the private consumption marginal utility: $(1-\rho)v'(\tau e) \geq \rho$. Finally, we choose a neutral value for consumption preference $\rho = 1/2$ for the baseline exercises. We will use this parameter to obtain some static comparative results later.

The parameter κ of the inflation disutility function is set according to Campos and Cysne (2018)'s estimation of a 0.35% of GDP cost for inflation of 10% in recent Brazilian experience, which we adjust to disutility by calculating the utility loss. The fixed cost of deviating ϵ equals 0.004, meaning a permanent 0.27% of GDP penalty for deviating from the target, and it is set to jointly match the gross debt level for the FFZ and the inflation index observed in Brazil in 2002. The crisis zone starts at approximately 70% of the debt ratio, slightly below the debt observed in 2002. Table I summarizes the chosen values.

Insert Table I about here.

3.2 Results

An indebted and altruistic policymaker optimally choosing inflation may deviate from the target in the event of an expectation shock. In our calibrated model, the policymaker becomes subject to such shocks after reaching a debt-to-GDP ratio of 70%. Below 70% of the debt ratio, the policymaker always prefers to keep inflation on target. For debt levels exceeding this lower bound, the equilibrium depends on the private agents' expectations, and the policymaker may decide to deviate given a negative sunspot shock. Taking this probability into account, private agents will demand higher nominal interest rates on government bonds once the policymaker exceeds this lower bound debt level. Finally, for debt levels exceeding 95% of GDP, the policymaker will always deviate from the target.

Optimal Fiscal Policy

The policymaker's optimal debt path depends upon the initial value of its debt stock. Outside the FFZ, it prefers to maintain debt levels constant, as shown in Proposition 2. Within the FFZ, it might i) choose fiscal responsibility and run down its debt to avoid the costs of an adverse equilibrium; ii) maintain constant debt levels; or iii) increase its debt to maintain a given spending level. In Figure II, we plot the next period's debt as a function of current debt. The three possible responses of the policymaker are seen within the FFZ. Those results are similar to those of Cole and Kehoe (1996).

Insert Figure II about here.

For a moderate initial debt level within the FFZ, the policymaker chooses a fiscally responsible debt path to avoid the expected endowment loss from deviating from the inflation target in the eventuality of a negative sunspot. In this region, expected inflation is higher than the target rate, which means that the policymaker faces a higher real interest rate than in the no crisis zone. However, as long as a negative sunspot shock that removes the credibility of the policymaker does not hit the economy, the optimal fiscal policy is to gradually reduce the debt-to-GDP ratio until the economy exits the FFZ. As the policymaker follows this austerity policy, expected inflation gradually declines, reducing the real interest rate burden and making it easier for the economy to exit the FFZ. This can be noted by observing that the slope of the policy function decreases as debt-to-GDP approaches the lower bound of the FFZ. Table II presents the expected inflation rates and the corresponding number of periods required to exit the FFZ for different levels of initial debt:

Insert Table II about here.

For a high initial debt level and fixed inflation target, the policymaker gradually reduces debt to return to the no-crisis zone. However, it takes a significant number of periods for the policymaker to regain credibility, during which it faces expected inflation rates higher than the target. The optimal policy for stabilizing inflation expectations in an environment of high indebtedness results in higher inflation expectations for a significant amount of time. Gradual exiting as an optimal decision is also present in Cole and Kehoe (1996, 2000) and in Sims (2020). The latter explicitly makes this prescription when claiming that it is not wise to reduce debt quickly by contracting fiscal latitude. This result is in line with many episodes in which countries that experienced sudden increases in their debt-to-GDP ratios needed to stabilize their economies through higher temporary inflationary rates. Hall and Sargent (2022) describe the US post-war experience and attribute an important role in reducing the real value of debt to increases in price levels. They argue that a similar scenario may happen after the rise in the debt-to-GDP ratio that followed the COVID-19 pandemic.

Nevertheless, as the debt level grows, the fiscal room available to the policymaker shrinks due to the increased interest burden. Eventually, it is more desirable to run up debt to maintain spending. This situation happens above debt levels of 89%, as seen in Figure II. An austerity policy to exit the FFZ is not optimal, and the policymaker eventually suffers an adverse shock and loses credibility. By opting to run up debt, the policymaker will ultimately fail to give the needed fiscal support to the inflation target.

Coordinating Expectations Through the Target

Higher inflation targets may improve the credibility of monetary policy and help coordinate private agents' expectations by increasing the costs of deviating to attain a given inflationary transfer of resources. Private agents use the inflation target to form expectations in the FFZ, and the target functions as a nominal anchor for expectations. In Figure III, we present the results of a sensitivity analysis of the deviations to changes in the inflation target. We plot $\pi^D - \pi^a$ for three different inflation targets (2%, 3.5%, and 5%), keeping the other parameters at their baseline.

Insert Figure III about here.

A higher inflation target improves coordination by the policymaker by reducing the discretionary deviation from the target rate and by reducing the real fiscal burden of debt through a lower real interest rate. A higher target rate reduces the marginal capacity of the policymaker to transfer resources, which implies a lower marginal benefit from discretionary inflation. As a consequence, a policymaker with a higher inflation target faces a lower real interest rate on its bonds in the FFZ, as shown in Proposition 4 and chooses a smaller deviation from the target.

For baseline parameters, deviations $\pi^D - \pi^a$ decrease in the inflation target, reducing the ex post real interest rate in the FFZ. For initial debt levels below the lower bound of the FFZ, $\underline{D}$, the policymaker will have a perfectly credible target, preferring to keep inflation on target regardless of private agent expectations. Above the lower bound, private agents may doubt its commitment. As deviations decrease in the target, it becomes less costly to keep inflation on target for a given debt level. This effect increases the credibility of the inflation target because it remains fully assured up to higher levels of debt, as shown in Proposition 5. In Figure IV we plot next-period debt for different inflation targets.

Insert Figure IV about here.

The lower bound $\underline{D}$ increases as the target rate raises. This result implies that policymakers should consider current debt levels and fiscal conditions when deciding to decrease the inflation target. This reduction can cause a loss of credibility for the government commitment as it enters the FFZ. While in the FFZ, expected inflation is higher than the target inflation rate, and choosing a low inflation target is costly instead of optimal. This result is in line with Araujo, Berriel, and Santos (2016), where a lower inflation target might reduce the policymaker's coordination ability due to a loss of credibility in its commitment and result in a worse equilibrium outcome.

The above analysis suggests a tradeoff when defining the target inflation rate for an economy with poor fiscal conditions. A lower target means a reduced welfare cost of inflation in the no-crisis zone and reduced discretionary inflation in the fiscal fragility and dominance zones, which is desirable for the policymaker. However, reducing the target also causes lower debt levels to be in the FFZ, which substantially reduces welfare since there is a positive probability that a sunspot shock that causes a permanent credibility loss will hit the economy. By computing the inflation target that maximizes total intertemporal utility for each initial debt level, we see that the optimal target is the lowest target possible such that the current debt level is in the no-crisis zone, as shown in Table III:

Insert Table III about here.

This result shows that there is a rationale for raising the inflation target in countries in a fragile fiscal situation, even when we consider the existence of costs associated with higher inflation levels. It is optimal for the policymaker to raise the target until the economy exits the FFZ. In this model, since the target is a fixed parameter, the optimal policy outside of the FFZ will be stationary, and it will not be optimal to further reduce the debt level. However, we may conjecture that in a scenario in which the inflation target is treated as a policy instrument, raising the target will be temporary, and further austerity to reduce debt levels even in the no-crisis zone will be optimal to support a lower inflation target in future periods. This is best understood as *gradual disinflation* for economies in a fragile fiscal situation.

Inflation-Indexed Debt

Indexed debt was defined by taking away the uncertainty about which equilibrium would be selected next period and, consequently, revealing the sunspot variable to private agents. As a result, private agents are able to correctly anticipate inflation and obtain a constant real interest rate on their bond holdings. We show that indexed debt, so defined, comes with higher inflation.

Recall that we find discretionary inflation by solving for the discretionary policymaker's optimal inflation given expectations; that is, given π^e, we find the optimal π^D. The difference between the two timing assumptions is in the formation of inflation expectations. In the FFZ with nominal debt, the inflation expectation π^e is equal to $f\pi^D+(1-f)\pi^a$. Agents form expectations accounting for the probability of the policymaker delivering the target. With indexed debt, the inflation expectation π^e is equal to π^D. Agents do not form expectations accounting for the probability of the policymaker delivering the target. Intuitively, the policymaker attempts to transfer resources. However, it is unable to use inflation to partially default when subjected to a negative expectations shock since private agents adapt their expectations. This dynamic leads to higher levels of discretionary inflation. The optimal inflation chosen by the policymaker when its debt stock is higher nominal versus inflation-indexed is depicted in Figure V.

Insert Figure V about here.

The higher discretionary inflation resulting from this timing may change the credibility of the inflation target for an initial debt stock, as the cost of maintaining the target increases in discretionary inflation under adverse expectations. Debt levels D in the no-crisis zone support the inflation target with certainty. Higher levels of discretionary inflation imply a higher penalty when deviating, and given that with indexed debt we have that $\pi^e = \pi^D$, the stationary level of public spending is lower when deviating, since the policymaker is not able to reduce interest spending. As a consequence, there is a higher penalty when deviating from the target when the government is financed by indexed bonds. This makes it more beneficial for the policymaker to commit to the target and avoid the crisis zone. As a result, the lower bound of the FFZ $\underline{\mathrm{D}}$ increases when debt is indexed, indicating a higher credibility of the target because a greater set of initial debt levels fully supports it. However, for high debt levels in which the target is not supported, inflation is higher and the economy enters a steady-state associated with a higher inflation level, and therefore higher costs in terms of welfare than in the case with prefixed bonds.

Preference for Spending

A shock to preferences can connect our model to the situation observed in Brazil during the 2002 confidence crisis. Suppose that policymaker preferences shift toward giving more weight to public spending. Decreasing ρ would be tantamount to increasing the weight of public spending. This shift changes marginal utilities and the optimal allocation of resources, increasing the share going to public spending. The altruistic policymaker chooses higher discretionary inflation levels.

Insert Figure VI about here.

Given a debt level, a relatively higher preference for public spending increases the level of discretionary inflation. For initial debt, a preference shock could push the policymaker into the FFZ. A sufficiently large shock to ρ could result in the loss of credibility of the target under adverse expectations. Private agents would adapt their inflation expectations. A non-null probability assigned to an adverse event would increase expectations compared to a scenario where the target is perfectly assured. Such a preference shock explains how expectations can suddenly exceed the target, as happened in Brazil in 2002.

4 2002 Confidence Crisis in Brazil

In 2002 and 2003, Brazilian policymakers faced inflationary pressures when it became clear that the left-wing presidential candidate would win. The perception was that his victory would mean implementing a new policy framework that could undermine the previous inflation reduction. Consequently, inflation expectations overshot the target's upper bounds at all horizons relevant to the central bank, as shown in Figure I. We map this event in our model as a shock to the preference for spending in the parameter ρ. Sensitivity analysis in Section 3.2 shows that, for a given initial debt stock, the target could lose credibility after a preference shock. By favoring more public spending, the policymaker could become vulnerable to adverse shocks that would make it deviate from the inflation target. Private agents taking this probability into account when forming expectations would increase their forecasts of future inflation, precisely as observed in 2002.

In response to rising inflation expectations, the outgoing and new administrations took several steps. First, to coordinate inflation expectations in the short run, they increased the target for 2003 in an additional meeting held in June 2002 and unofficially again in January 2003. Second, during 2003, public debt reduction sustained responsible macroeconomic policies. Ultimately, inflation expectations converged back to the target. These policy responses closely mirror the prescriptions suggested by our model. We will consider each of these policies in further detail.

Fiscal Policy

After the 2002 election, the government gradually reduced the gross public debt. The gross debt declined from nearly 80% of GDP in 2002 to nearly 70% in 2004. Furthermore, the government continued to run primary surpluses to meet its debt obligations in a signal of fiscal responsibility. The primary surplus increased from 2.16% of GDP in 2001 to 2.70% in 2004. From the perspective of our model, such fiscal policy is compatible with the policymaker attempting to exit the FFZ and give the needed fiscal support to its inflation target.

Inflation Target

Before the October elections, the 2003 target was exceptionally revised upward from a previously announced 3.25% to 4%. Similarly, the upper and lower bounds widened from -/+ 2% to 2.5%. In January 2003, the Ministry of Finance sent a letter stating that the adjusted target would be 8.5% in 2003 and 5.5% in 2004. The latter was confirmed by the National Monetary Committee as the inflation target for 2004 in June 2003, as we can see in Table IV. From the perspective of our model, an indebted economy with a higher inflation target might be more credible. The higher and more credible inflation target serves as a nominal anchor, making private agents readjust their inflation expectations.

Insert Table IV about here.

5 Concluding Remarks

In this paper, we develop a model to study the problem of setting the inflation target in an economy with high debt burden. In light of our results, we are able to analyze a concrete case in which the Brazilian economy was subject to an adverse inflation expectations shock, and where the policymakers were successfully able to avoid a severe loss of credibility by simultaneously raising the inflation target and adopting fiscal austerity measures. Our results are consistent with the observed behavior of Brazilian policymakers and its consequences, and provide practical implications for the design of the monetary policy framework for economies with fragile fiscal fundamentals. Adopting a low inflation target may not be welfare improving if it raises doubts over the commitment of the policymaker to provide the necessary fiscal backing for delivering the target.

Appendix A Proofs

A.1 Discretionary Inflation is Increasing in the Debt Level

Suppose that π_T^D is an interior solution to the discretionary inflation problem. To avoid a cumbersome notation we drop the superscript on r_T^D and let

$$1 + r_T = \frac{1}{\beta} \frac{1}{f + (1-f)\frac{1+\pi}{1+\pi^a}} \tag{16}$$

be the real interest rate when the policymaker deviates to inflation level π, where f is the probability of a negative sunspot shock. To obtain $\partial \pi_T^D / \partial D_T$, we differentiate the first-order condition (11) with respect to the initial debt level D_T to obtain:

$$\begin{aligned} -(1-\rho)v''(g)\frac{\partial g}{\partial D_T}\frac{\partial(1+r_T)}{\partial \pi}D_T + (\rho - (1-\rho)v'(g))\frac{\partial^2(1+r_T)}{\partial \pi \partial D_T}D_T \\ +(\rho - (1-\rho)v'(g))\frac{\partial(1+r_T)}{\partial \pi} = \frac{1}{1-\beta}\psi''(\pi)\frac{\partial \pi}{\partial D_T} \end{aligned} \tag{17}$$

where

$$\frac{\partial g}{\partial D_T} = -\beta \frac{\partial(1+r_T)}{\partial \pi}\frac{\partial \pi}{\partial D_T}D_T - \beta(1+r_T) \tag{18}$$

and

$$\frac{\partial^2(1+r_T)}{\partial \pi \partial D_T} = \frac{\partial^2(1+r_T)}{\partial \pi^2}\frac{\partial \pi}{\partial D_T} \tag{19}$$

since $1 + r_T$ does not depend directly on D_T.

To prove that $\partial \pi_T^D / \partial D_T > 0$, we substitute back equations (18) and (19) in (17) and rearrange to obtain:

$$\begin{aligned} \left[\frac{1}{1-\beta}\psi''(\pi) - \beta(1-\rho)v''(g)\left(\frac{\partial(1+r_T)}{\partial \pi}\right)^2 D_T^2 - [\rho - (1-\rho)v'(g)]\frac{\partial^2(1+r_T)}{\partial \pi^2}\right]\frac{\partial \pi}{\partial D_T} \\ = [\rho - (1-\rho)v'(g) + \beta(1-\rho)v''(g)(1+r_T)D_T] \end{aligned} \tag{20}$$

By differentiating (16) it is easy to conclude that $\frac{\partial(1+r_T)}{\partial \pi} < 0$ and $\frac{\partial^2(1+r_T)}{\partial \pi^2} > 0$. Also, by hypothesis, we have that for every feasible g, $\rho - (1-\rho)v'(g) < 0$ and $v''(g) < 0$. Since both the term multiplying $\frac{\partial \pi}{\partial D_T}$ and the term on the right hand side in equation (20) are strictly positive, we conclude that $\frac{\partial \pi}{\partial D_T} > 0$.

A.2 Optimal Debt Policy Outside the Fiscal Fragility Zone

Outside of the FFZ, there is only a unique inflation equilibrium, making it perfectly anticipated. The policymaker's problem can be reduced to the following:

$$\begin{aligned}\max_{D_{t+1}} \sum_{t \geq S} & \beta^t u(c_t, g_t) \\ \text{s.t. } c_t &= \frac{1}{\beta} D_t + (1-\tau)(e-\epsilon^*) - D_{t+1} \\ g_t &= D_{t+1} + \tau(e-\epsilon^*) - \frac{1}{\beta} D_t\end{aligned}$$

where ϵ^* is equal to 0 if the economy is in the no-crisis zone and equal to ϵ if it is in the fiscal dominance zone. The first-order condition (FOC) for D_{t+1} yields:

$$(1-\rho)v'(g_t) - \rho = (1-\rho)v'(g_{t+1}) - \rho$$

which implies $v'(g_t) = v'(g_{t+1})$ for all t. Given that v strictly concave in g, we must have $g_{t+1} = g_t$. Replacing g_t and g_{t+1} by the government budget equation, iterating forward and taking limits, we obtain:

$$\lim_{t \to \infty} D_{S+t} = \sum_{i=1}^{\infty} \left(\frac{1}{\beta}\right)^i (D_{S+1} - D_S) + D_{S+1}. \tag{21}$$

Suppose that $D_{S+1} \neq D_S$; then, the policymaker will either run up infinite debt or credit. Equation (21) also implies that

$$\lim_{t \to \infty} \beta^{S+t-1} D_{S+t} = \beta^S \sum_{i=0}^{\infty} \beta^i (D_{S+1} - D_S) + \lim_{t \to \infty} \beta^{S+t-1} D_{S+1} = \frac{\beta^S}{1-\beta}(D_{S+1} - D_S).$$

The no-Ponzi condition for this problem states that

$$\lim_{t \to \infty} \beta^t D_{t+1} = 0,$$

so that if $D_{S+1} \neq D_S$, this condition is violated. This means that the only optimal trajectory for debt outside of the FFZ is the stationary state such that $D_t = D_S = D_{S+1}$ for all t.

A.3 Above-Target Discretionary Inflation

The intuition for the proof is simple: we show that if it is the case that $\pi^D < \pi^a$, then there exists a feasible policy that does not deviate from the target and attains a higher intertemporal utility then deviating, even if private agents believe that the policymaker will deviate from target. This implies that the economy is in the no-crisis zone. Therefore, whenever the economy is in the FFZ, deviation from target must be positive.

For a given debt level D, assume that $\pi^D < \pi^a$, that is, the discretionary inflation rate is lower than the target rate. We want to prove that in this case, $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)$, that is, the policymaker follows the inflation target even when the private agents expect the policymaker to deviate. Let T be the time of deviation, and assume that private agents expect the policymaker to deviate; then, according to Equation (7), total government spending both in period T and in the stationary long-run will be equal to

$$g^D = \tau(e - \epsilon) - \left(\frac{1}{\beta} - 1\right) D, \tag{22}$$

since agents expect the deviation.

As an alternative, the government can choose the feasible path of not deviating from the target and following a stationary spending policy:

$$g^a = \tau e - \left(\frac{1 + \pi^D}{1 + \pi^a} \frac{1}{\beta}\right) D. \tag{23}$$

Since $\pi^D < \pi^a$, we have that $g^D < g^a$, as

$$g^a - g^D = \tau\epsilon + \frac{\pi^a - \pi^D}{1 + \pi^a} \frac{1}{\beta} D > 0.$$

To compare the total intertemporal utility of the two policies, we only need to compare which one of the allocations achieves a higher utility in any period, since they are stationary allocations. Let $c^D = e - \epsilon - g^D$ and $c^a = e - g^a$ be the market-clearing private consumption in each scenario. By the concavity of the utility function, we have that

$$\begin{aligned} u(c^D, g^D) - u(c^a, g^a) \leq & \rho(c^D - c^a) + (1 - \rho)v'(g^a)(g^D - g^a) \\ = & -\rho\epsilon + ((1 - \rho)v'(g^a) - \rho)(g^D - g^a) \\ < & -\psi(\pi^a) < \psi(\pi^D) - \psi(\pi^a) \end{aligned} \tag{24}$$

since, by the assumption, we have that $(1 - \rho)v'(g^a) - \rho \geq 0$, $\rho\epsilon > \psi(\pi^a)$, and $\pi^D \leq \pi^a$.

Now, since there is a feasible policy trajectory in which the policymaker follows the inflation target and its intertemporal utility is greater than that attained by deviating to the inflation rate π^D, this means that the optimal policy chosen by the policymaker when following the target must also attain a higher utility than the attained by deviating, that is, $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)$. However, this means that the policymaker chooses to follow the target even when private agents expect it to deviate, so that debt level D is in the no-crisis zone.

A.4 Real Interest Rates in the FFZ when Raising the Target

By the definition of the real interest rate, when the policymaker follows the target in the FFZ we have

$$1 + r_t^a = \frac{1}{\beta} \frac{1}{f \frac{1+\pi^a}{1+\pi} + 1 - f}. \tag{25}$$

Differentiating with respect to π^a and we obtain

$$\frac{\partial(1 + r_t^a)}{\partial \pi^a} = \frac{1}{\beta} \frac{1}{\left[f \frac{1+\pi^a}{1+\pi} + 1 - f\right]^2} \frac{f}{1+\pi} \left(\frac{1+\pi^a}{1+\pi} \frac{\partial \pi}{\partial \pi^a} - 1 \right) \tag{26}$$

so that the desired result is shown if the term in parenthesis is positive. To prove that, we calculate $\partial \pi / \partial \pi^a$ implicitly by differentiating equation (11)[18] with respect to π^a, obtaining:

$$(1-\rho)v''(g)(1-\beta)\frac{\partial(1+r_t)}{\partial \pi^a}\frac{\partial(1+r_t)}{\partial \pi}D_T^2 + (\rho-(1-\rho)v'(g))\frac{\partial^2(1+r_t)}{\partial \pi \partial \pi^a}D_T = \frac{1}{1-\beta}\left[\psi''\frac{\partial \pi}{\partial \pi^a} + \frac{\partial \psi'}{\partial \pi^a}\right] \tag{27}$$

From the definition of $1 + r_t$, we can obtain the following identities:

$$\begin{aligned}
\frac{\partial(1+r_t)}{\partial \pi^a} &= \frac{\partial(1+r_t)}{\partial \pi}\left[\frac{\partial \pi}{\partial \pi^a} - \frac{1+\pi}{1+\pi^a}\right] \\
\frac{\partial^2(1+r_t)}{\partial \pi \partial \pi^a} &= \frac{\partial(1+r_t)}{\partial \pi}\left[2\beta\frac{1-f}{1+\pi^a}(1+r_t)\left(\frac{1+\pi}{1+\pi^a} - \frac{\partial \pi}{\partial \pi^a}\right) - \frac{1}{1+\pi^a}\right]
\end{aligned}$$

[18] We again drop the superscript on r_T^D to avoid cumbersome notation.

which we can substitute back to find the solution for $\partial\pi/\partial\pi^a$, given by:

$$\left[2\beta\frac{1-f}{1+\pi^a}(1+r_t)(\rho-(1-\rho)v'(g))D_T\frac{\partial(1+r_t)}{\partial\pi}-(1-\beta)(1-\rho)v''(g)D_T^2\left(\frac{\partial(1+r_t)}{\partial\pi}\right)^2\right.$$
$$\left.+\frac{1}{1-\beta}\psi''\right]\frac{\partial\pi}{\partial\pi^a}=-\frac{1}{1-\beta}\frac{\partial\psi'}{\partial\pi^a}-(1-\beta)(1-\rho)v''(g)D_T^2\left(\frac{\partial(1+r_t)}{\partial\pi}\right)^2\frac{1+\pi}{1+\pi^a}$$
$$+(\rho-(1-\rho)v'(g))D_T\frac{\partial(1+r_t)}{\partial\pi}\left[2\beta\frac{1-f}{1+\pi^a}(1+r_t)\frac{1+\pi}{1+\pi^a}-\frac{1}{1+\pi^a}\right]$$

The first term in brackets is clearly strictly positive so that $\partial\pi/\partial\pi^a$ is well defined. Using the result above we can calculate the term in parenthesis in equation (26), which results in a fraction with the same (strictly positive) denominator as $\partial\pi/\partial\pi^a$, and numerator given by:

$$-\frac{1}{1-\beta}\left[\psi''+\frac{1+\pi^a}{1+\pi}\frac{\partial\psi'}{\partial\pi^a}\right]-(\rho-(1-\rho)v'(g))\frac{\partial(1+r_t)}{\partial\pi}\frac{1}{1+\pi}D_T<0$$

which proves that $\frac{\partial(1+r_t^a)}{\partial\pi^a}<0$. An analogous calculation shows that $\frac{\partial(1+r_t^D)}{\partial\pi^a}>0$, and we are done.

A.5 Lower Limit of the Fiscal Fragility Zone

We start by characterizing the debt level $\underline{D}$ that is the lower limit of the FFZ. Consider a D in the no-crisis zone, then:

$$V^p(D,\pi_t=\pi^a,\pi^e=\pi^D)=u\left((1-\tau)e+\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)D,\tau e-\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)D\right)-\psi(\pi^a)$$
$$+\beta V^p(D,\pi^e,\zeta=0,\epsilon_{-1}=0)$$

Since for every debt level in the no-crisis zone it is optimal to keep the debt level constant in every period by Proposition 2, we know that $\pi^e=\pi^a$ and

$$V^p(D,\pi^e,\zeta=0,\epsilon_{-1}=0)=\frac{1}{1-\beta}\left[u\left((1-\tau)e+\left(\frac{1}{\beta}-1\right)D,\tau e-\left(\frac{1}{\beta}-1\right)D\right)-\psi(\pi^a)\right]$$

Then, for such D, the no-crisis zone condition is satisfied with

$$\begin{aligned}
V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) =& u\left((1-\tau)e + \left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a} - 1\right)D, \tau e - \left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a} - 1\right)D\right) - \psi(\pi^a) \\
&+ \frac{\beta}{1-\beta}\left[u\left((1-\tau)e + \left(\frac{1}{\beta} - 1\right)D, \tau e - \left(\frac{1}{\beta} - 1\right)D\right) - \psi(\pi^a)\right] \\
\geq& \frac{1}{1-\beta}\left[u\left((1-\tau)(e-\epsilon) + \left(\frac{1}{\beta} - 1\right)D, \tau(e-\epsilon) - \left(\frac{1}{\beta} - 1\right)D\right) - \psi(\pi^D)\right] \\
=& V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)
\end{aligned}$$

Since π^D is continuous with respect to D, we have that in the point $\underline{\text{D}}$ the condition above is satisfied with equality, otherwise there would exist a $D > \underline{\text{D}}$ sufficiently close that satisfies the above condition and so is also in the no-crisis zone. Let $\underline{\text{D}}$ be the point where the condition is satisfied with equality, so that

$$\begin{aligned}
&u\left((1-\tau)e + \left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a} - 1\right)\underline{\text{D}}, \tau e - \left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a} - 1\right)\underline{\text{D}}\right) - \psi(\pi^a) \\
&+ \frac{\beta}{1-\beta}\left[u\left((1-\tau)e + \left(\frac{1}{\beta} - 1\right)\underline{\text{D}}, \tau e - \left(\frac{1}{\beta} - 1\right)\underline{\text{D}}\right) - \psi(\pi^a)\right] \\
&= \frac{1}{1-\beta}\left[u\left((1-\tau)(e-\epsilon) + \left(\frac{1}{\beta} - 1\right)\underline{\text{D}}, \tau(e-\epsilon) - \left(\frac{1}{\beta} - 1\right)\underline{\text{D}}\right) - \psi(\pi^D)\right]
\end{aligned}$$

We can rewrite the above expression as

$$u\left((1-\tau)e + \left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a} - 1\right)\underline{\text{D}}, \tau e - \left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a} - 1\right)\underline{\text{D}}\right) - \frac{1}{1-\beta}\left[\psi(\pi^a) - \psi(\pi^D)\right] \tag{28}$$

$$\begin{aligned}
=&\frac{1}{1-\beta}u\left((1-\tau)(e-\epsilon) + \left(\frac{1}{\beta} - 1\right)\underline{\text{D}}, \tau(e-\epsilon) - \left(\frac{1}{\beta} - 1\right)\underline{\text{D}}\right) \\
-&\frac{\beta}{1-\beta}u\left((1-\tau)e + \left(\frac{1}{\beta} - 1\right)\underline{\text{D}}, \tau e - \left(\frac{1}{\beta} - 1\right)\underline{\text{D}}\right)
\end{aligned}$$

in which only the left-hand side varies when we change the inflation target π^a. We now prove that the left-hand side of the above expression is increasing in π^a, which implies that for a higher target the no-crisis zone condition is satisfied with inequality for $\underline{\text{D}}$. This means that there exists a $D > \underline{\text{D}}$ in the no crisis zone, and therefore the lower limit of the fiscal fragility zone is higher under the higher inflation target.

Differentiate the left hand side of equation (28) with respect to π^a to obtain

$$LHS'(\pi^a) = (\rho - (1-\rho)v'(g_a^D))\frac{1}{1+\pi^a}\left(\frac{\partial \pi^D}{\partial \pi^a} - \frac{1+\pi^D}{1+\pi^a}\right)\underline{\mathrm{D}} - \frac{1}{1-\beta}\left(\psi'(\pi^a) - \psi'(\pi^D)\frac{\partial \pi^D}{\partial \pi^a}\right) \tag{29}$$

where $g_a^D = \tau e - \left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a} - 1\right)\underline{\mathrm{D}}$ is government spending when delivering the target given expectations to deviate. We substitute the expression for $\frac{\partial \pi^D}{\partial \pi^a}$ from the proof of Proposition 4, to obtain

$$\begin{aligned}
&\left[\frac{1}{1-\beta}\psi''(\pi^D) + 2\beta\frac{1-f}{1+\pi^a}(1+r)(\rho-(1-\rho)v'(g))\underline{\mathrm{D}}\frac{\partial(1+r)}{\partial \pi}\right.\\
&\qquad \left. -(1-\beta)(1-\rho)v''(g)\underline{\mathrm{D}}\left(\frac{\partial(1+r)}{\partial \pi}\right)^2\right]LHS'(\pi^a)\\
&= \left[\frac{1+\pi^D}{1+\pi^a}\psi'(\pi^D) - \psi'(\pi^a)\right]\left(2\frac{\beta}{1-\beta}\frac{1-f}{1+\pi^a}(1+r)(\rho-(1-\rho)v'(g))\underline{\mathrm{D}}\frac{\partial(1+r)}{\partial \pi}\right.\\
&\qquad \left. -(1-\rho)v''(g)\underline{\mathrm{D}}^2\left(\frac{\partial(1+r)}{\partial \pi}\right)^2\right)\\
&- \left((\rho-(1-\rho)v'(g))\frac{\underline{\mathrm{D}}}{1+\pi^a}\frac{\partial(1+r)}{\partial \pi} + \frac{1}{1-\beta}\frac{\partial \psi'}{\partial \pi^a}\right)\left[(\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)} + \frac{1}{1-\beta}\psi'(\pi^D)\right]\\
&\qquad - \frac{1}{1-\beta}\psi''(\pi^D)\frac{1+\pi^D}{1+\pi^a}\left[(\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)} + \frac{1}{1-\beta}\psi'(\pi^a)\frac{1+\pi^a}{1+\pi^D}\right]
\end{aligned} \tag{30}$$

where g is government spending solution and r is the real interest rate in the optimal deviation problem. The sign of $LHS'(\pi^a)$ is determined by the sign of the term in the right hand side of (30), since as in the proof of Proposition 4 the denominator of $\frac{\partial \pi^D}{\partial \pi^a}$ is positive.

We start by showing that

$$\begin{aligned}
&\left[\frac{1+\pi^D}{1+\pi^a}\psi'(\pi^D) - \psi'(\pi^a)\right]\left(2\frac{\beta}{1-\beta}\frac{1-f}{1+\pi^a}(1+r)(\rho-(1-\rho)v'(g))\underline{\mathrm{D}}\frac{\partial(1+r)}{\partial \pi}\right.\\
&\qquad \left. -(1-\rho)v''(g)\underline{\mathrm{D}}^2\left(\frac{\partial(1+r)}{\partial \pi}\right)^2\right) > 0
\end{aligned}$$

Using the same analysis as in the proofs of Propositions 1 and 4 we know that the term in parenthesis is strictly positive. We also have that

$$\frac{1+\pi^D}{1+\pi^a}\psi'(\pi^D) - \psi'(\pi^a) > 0$$

is immediate from the hypothesis that $\psi'' \geq 0$, and the fact that $\pi^D > \pi^a$. This is true since we know from Proposition 3 that if $\pi^D \leq \pi^a$ then the no-crisis zone condition is satisfied with strict inequality, and in point $\underline{\mathrm{D}}$ it is satisfied with equality.

We now prove that the rest of the right hand side of 30 is positive. The previous reasoning also imply that

$$(\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}+\frac{1}{1-\beta}\psi'(\pi^a)\frac{1+\pi^a}{1+\pi^D} < (\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}+\frac{1}{1-\beta}\psi'(\pi^D)$$

so that we only need to show that

$$\begin{aligned}-\left((\rho-(1-\rho)v'(g))\frac{\underline{\mathrm{D}}}{1+\pi^a}\frac{\partial(1+r)}{\partial\pi}+\frac{1}{1-\beta}\left(\frac{1+\pi^D}{1+\pi^a}\psi''(\pi^D)+\frac{\partial\psi'}{\partial\pi^a}\right)\right)&\\ \left[(\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}+\frac{1}{1-\beta}\psi'(\pi^D)\right]&>0\end{aligned}$$

Again, the fact that the term in parenthesis is strictly positive was proved in Proposition 4. We only need to prove that the term in braces is strictly negative. Using the first order condition (11) and the definition for $1+r$ we get

$$\begin{aligned}\frac{1}{1-\beta}\psi'(\pi^D) =&(\rho-(1-\rho)v'(g))\frac{\partial(1+r)}{\partial\pi}\underline{\mathrm{D}}\\ =&((1-\rho)v'(g)-\rho)\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}\frac{(1-f)}{\left(f+(1-f)\frac{1+\pi^D}{1+\pi^a}\right)^2}\\ <&((1-\rho)v'(g_a^D)-\rho)\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}\end{aligned}$$

so long as we show that $g_a^D < g$, since $0 < \frac{(1-f)}{\left(f+(1-f)\frac{1+\pi^D}{1+\pi^a}\right)^2} < 1$. But from expression (28), we know that

$$u(e-g_a^D,g_a^D)+\frac{\beta}{1-\beta}u(e-g_a^a,g_a^a)-\frac{1}{1-\beta}\psi(\pi^a)=\frac{1}{1-\beta}u(e-\epsilon-g_D^D,g_D^D)-\frac{1}{1-\beta}\psi(\pi^D)$$

where $g_a^a=\tau e-(\frac{1}{\beta}-1)\underline{\mathrm{D}}$ and $g_D^D=\tau(e-\epsilon)-(\frac{1}{\beta}-1)\underline{\mathrm{D}}$. Since $g_a^a > g_D^D$ and $\psi(\pi^D) > \psi(\pi^a)$, the only way for the above equality to be satisfied is for $g_a^D < g_D^D$, since u is increasing in government spending. However, it is straightforward to show that $g_D^D < g$, since g is the stationary spending when the government is able to partially default in debt and reduce interest spending. But this proves that $g_a^D < g$ and we are done.

Appendix B Empirical Results

The calibrated model leads to the conclusions that i) the size of the deviation could be reduced by increasing the target and reducing debt and ii) the probability of deviating from the target would increase with debt and decrease with higher target levels. The present section investigates whether there is empirical evidence for the predictions based on our model. We construct a dataset that includes 20 countries with at least 15 years of inflation targeting[19] covering the period from 2000 to 2019. Targets are those reported by the respective central banks that were manually collected from each central bank web page. Inflation and gross debt and revenue to GDP statistics are from the IMF. With regard to inflation, end-of-year consumer price inflation is the target benchmark. Some general statistics are reported in Table B.I. The variables present both inter- and intracountry variability. In the case of CPI targets, 55% of our sample changed the target at least once. Most of the changes are in middle-income countries.[20]

Insert Table B.I about here.

Real effective exchange rate (Reer) and GDP gap estimates enter robustness checks. When Reer statistics were not available from the IMF, other sources were accessed.[21] GDP gap estimates are constructed using quarterly seasonally adjusted GDP volume statistics from the IMF. When not available, the unadjusted equivalents are seasonally adjusted with the Arima X-11 procedure.[22] The quarterly GDP gap statistics are obtained applying an HP filter with a smoothing parameter of 1600. To mitigate the endpoint bias of the filter at the beginning of each series, we estimate the gap for the longer 1996Q1 - 2020Q1 period. Finally, the yearly GDP gap is defined as the average gap over the relevant period.

Deviations from the Target

The FOC of the discretionary inflation problem from 7 relates the deviation of inflation $\pi_{i,t}$ from the inflation target $\pi^a_{i,t}$ to observable and latent variables for each country i. We estimate the following model,

$$\pi_{i,t} - \pi^a_{i,t} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \pi^a_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{31}$$

[19] The countries in the sample are Australia, Brazil, Canada, Chile, Colombia, the Czech Republic, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, the Philippines, Poland, South Africa, Sweden, Thailand, Turkey, and the United Kingdom.

[20] We used the World Bank classification.

[21] BIS for Peru, Indonesia, and Turkey. Bank of Thailand for Thailand.

[22] This was the case for Peru and Turkey.

where the idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1},...,X_{i,T},c_i) = 0,\ t = 1,...,T$ with $X_{i,t}$ being a vector of the observable regressors at time t for country i. The variables and parameters of the model are mapped into both observed series and latent variables. We map the model variables D, τe, and π^a to gross debt (%GDP), revenue (%GDP), and the inflation target. The unobservable variables e, f, ϵ and κ are mapped into a country fixed effect c_i that captures the time-constant individual heterogeneity between countries. We use a fixed effect estimator because it seems reasonable to assume that their choices of debt, revenue and inflation target are related to the unobserved characteristics of each country c_i. In other words, we cannot assume $\mathbb{E}(X_{i,t}c_i) = 0\ \forall t$ as required for a random effect estimator.[23]

In terms of interpretation, the net impact of debt should be positive. Given higher levels of debt, the policymaker will have more incentive for discretionary inflation. Furthermore, discretionary inflation increases in debt. Hence, the deviation to increase in debt levels as the policymaker will be more likely to deviate and will choose higher discretionary inflation when doing so. Given an interaction term in (31), one would have to examine the joint impact captured by β_2 and β_4 for a given level of revenue to GDP. We also expect the coefficient on the inflation target to be negative because the policymaker could help coordinate private agents' expectations by adopting a more credible (higher) inflation target in given situations. Were inflation perfectly anchored, changing the target would not result in changes in expected deviation. In other words, the coefficient β_3 would equal zero. Finally, higher revenue means that the policymaker has more fiscal room for spending. This room decreases the incentives to transfer resources through discretionary inflation, leading to a negative net impact of revenue. Given the interaction term between debt and revenue, the joint impact captured by β_1 and β_4 should be negative for a given level of debt.

Estimation I in Table B.II is the basic model from (31). The remaining estimations, II-V, are robustness checks.

Insert Table B.II about here.

In estimation I, deviations from the target are on average negatively related to the target level. In the case of perfectly anchored inflation, the coefficient should not be significantly different from zero. We also have a positive coefficient on debt and a negative coefficient for the interaction term between debt and revenues. This can be interpreted as higher debt implying higher deviations for countries with limited revenues. For revenues no higher than 35% of GDP, the net impact of debt is positive. This result applies to the middle-income countries in our sample. The result goes in the direction of what the theoretical model predicted,

[23] A Hausman test between a fixed and random effect estimator similarly suggests the use of the former.

as both the probability of deviating and deviations from the target are positively related to debt levels. On average, countries with higher debt levels have higher deviations from their inflation target.

The coefficient on revenue is positive in all settings although not always significant. Given the interaction term with debt, the net impact of revenue is positive up to debt levels of 88%, above the maximum in our sample. Hence, the impact of higher revenue is to increase deviations from the inflation target. Although this goes against what was expected from the theoretical model, one could argue that higher revenue could be correlated with preferences for public spending that in turn could lead to inflationary pressure.

The results remain after accounting for different types of shocks and variables usually associated with inflation dynamics. In estimation II, we include a time fixed effect to account for global shocks such as commodity prices. In our sample, 2008 stands out, as many countries overshot their inflation targets after the financial crisis. The time dummies are meant to take such global comovements in inflation into account. Estimation III also includes shocks to the real effective exchange rate. Estimation IV adds the impact of deviations from potential GDP on inflation.

Probability of Deviating from the Target

The policymaker deviates from the inflation target when end-of-year inflation exceeds the upper bound of the target[24]. In the theoretical model, the policymaker had more incentive to deviate when it had limited fiscal space due to high debt servicing cost. We estimate a similar equation to (31) but with regard to the probability of deviating:

$$I_{\pi_{i,t} > \overline{\pi}^A_{i,t}} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{32}$$

where $\overline{\pi}^A_{i,t}$ is the upper bound of the inflation target for country i at time t. The indicator $I_{\pi_{i,t} > \overline{\pi}^A_{i,t}} = 1$ when inflation $\pi_{i,t}$ exceeds the upper bound of the inflation target $\overline{\pi}^A_{i,t}$. The idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0,\ t = 1, ..., T$. The probability of deviating will then be a logistic function:

$$Pr(I_{\pi_{i,t} > \overline{\pi}^A_{i,t}} = 1|X_{i,t}, c_i) = \frac{1}{1 + e^{-X'_{i,t}\beta - c_i}}, \quad t = 1, ..., T \tag{33}$$

[24] Some countries adopt pointwise targets instead of tolerance bounds. This is for instance the case for the UK and Norway. In such cases, we used the average upper tolerance limit from the rest of the sample (1.2%).

The expected results and dynamics are quite similar to those in the previous section with an expected net positive impact of debt, negative impact of the inflation target, and negative impact of revenue on the probability of deviating. Each year in the sample, at least two countries deviate from their respective inflation targets. The years 2007 and 2008 stand out, as over half of the countries deviated. A time dummy is likely to capture this effect. Additionally, virtually all countries except two deviated from their targets at least once, with some countries such as Turkey close to being serial deviators. Overall, middle-income countries exceed the target more often than high-income countries. Nevertheless, high-income countries exceeded the target 39 times.

The first column of Table B.III is the baseline model, while the remaining columns represent robustness checks similar in spirit to the previous section. When considering the net impact of debt on the probability of deviating from the target, the coefficients have similar signs to the previous estimates with regard to deviations from the target. Estimation I has the most restrictive condition for a net positive effect of debt. For revenues over 30% of GDP, the net effect of debt stops being positive. Not all middle-income countries in our sample have revenue below this level. However, the effects are not statistically significant in any of the settings.

Insert Table B.III about here.

The net impact of revenue remains positive for debt levels in the sample, not in the same direction as predicted by the theoretical model. The predicted negative impact of debt is based on increased fiscal room provided by higher revenue, which decreases the incentive to use discretionary inflation to transfer resources away from private agents' debt. However, another channel is possible. Revenue might be correlated with some other factors such as a higher preference for government spending, which would increase incentives to use inflation for transfer of resources. This channel could explain our results.

The probability of deviating is negatively related to the target level and significant at the 5% level in all settings. Our interpretation is that some countries might have inflation targets that are too low, making it more likely to exceed the target more often. Those counties could improve their ability to keep inflation on target by adopting higher targets. The results remain little changed when including shocks to exchange rates, the output gap, or a time dummy. Changes in the real effective exchange rate seem to be an important factor in causing policymakers to deviate. The output gap is not significant.

Appendix C Online Appendix: Productivity Cost of Inflation

In this section we present an alternative model in which inflation, instead of affecting utility directly, reduces the endowment of agents as inflation becomes higher. The rest of the model is identical, and results are nearly identical.

The policymaker maximizes an intertemporal utility with the same assumptions as the main model:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{34}$$

but is now subject to the following budget constraint:

$$g_t + (1 + r_t) D_t \leq D_{t+1} + \alpha_t \tau e \ , \tag{35}$$

The fixed endowment is subject to a penalty α_t that depends on the policymaker's choice of inflation. The penalty function α is divided into two components, α^p and α^c. The first component, α^p, depends on the inflation level and reflects the productivity cost of the inflation level on output. We assume a productivity cost of inflation function of the form of

$$\alpha^p(\pi) = (1 - \sigma) + \sigma e^{-\lambda \pi^2} \ ,$$

where $1 - \sigma$ is the lower limit on the inflation cost and λ is a fixed parameter. In this setup, $\alpha(0) = 1$, so the optimal inflation level considering only the productivity cost of inflation is zero. The second component is analogous to the credibility cost in the main model:

$$\alpha_t^c = \begin{cases} 0 & \text{if } \pi_t = \pi^a, \alpha_{t-1}^c = 0 \\ -\epsilon & \text{if } \pi_t \neq \pi^a, \alpha_{t-1}^c = 0 \\ \alpha_{t-1} & \text{otherwise} \end{cases}$$

We define $\alpha^a = \alpha^p(\pi^a)$, the productivity cost of committing to the inflation target. The optimal discretionary inflation problem can be rewritten as

$$
\begin{aligned}
&\max_{\pi,D} u(c_T, g_T) + \frac{\beta}{1-\beta} u(c,g) \\
&\text{subject to} \\
&\qquad g_T = \alpha(\pi)\tau e - (1+r_T^D)D_T + D \\
&\qquad g = \alpha(\pi)\tau e - D\left(\frac{1}{\beta} - 1\right) \\
&\qquad c_T = \alpha(\pi)e - g_T \\
&\qquad c = \alpha(\pi)e - g \\
&\qquad 1 + r_T^D = \frac{1}{\beta}\frac{1}{q_T^e}\frac{1}{1+\pi}.
\end{aligned}
\tag{36}
$$

The solution for the optimal deviation D is identical to the one given by equation 8, but the first-order condition for the discretionary inflation choice changes to

$$
(\rho - (1-\rho)v'(g))\frac{\partial(1+r_T^D)}{\partial\pi}D_T + (\rho(1-\tau) + (1-\rho)v'(g)\tau)\,\alpha' e = 0 \tag{37}
$$

which has the same intuition as before, but now raising the inflation level affects utility by reducing the available consumption in the steady-state, both for the private and the government consumption goods.

C.1 Calibration and Results

As before, we consider that $v(g) = log(g)$ and calibrate the parameters of the model following the same reasoning of the main model, but we now have an additional parameter σ to calibrate to match the 70% debt-to-GDP crisis zone observed in the Brazilian 2002 inflation crisis:

Insert Table C.I about here.

The main results are presented below. The policy function is nearly unchanged, as the calibration ensures that the crisis zone starts at around 70% of GDP:

Insert Figure C.I about here.

Expected inflation is lower for the model with productivity penalty, which imply a lower benefit of reducing debt rapidly and as a result the policymaker takes longer to exit the FFZ for moderate debt levels.

Insert Table C.II about here.

As before, the deviation from target is decreasing on the target level, and the inflation target serves the same coordinating role of expectations as the model with inflation disutility, raising the FFZ floor as the target is increased. Given the lower discretionary inflation, the optimal inflation target is slightly lower than the previous model.

Insert Figure C.II about here.

Insert Figure C.III about here.

Insert Table C.III about here.

Appendix D Online Appendix: Solution to Discretionary Inflation

Proposition 6 *Let the utility function $u(c, g)$ and the penalty function $\psi(\pi)$ be such that they satisfy the already stated assumptions. If the universe of possible inflation choices is defined on the compact set $[0, \overline{\pi}]$ where $\overline{\pi} > 0$ is some upper limit, then there exists a discretionary inflation level π^D such that π^D is optimal given private agents' inflation expectations π^e and vice versa.*

Proof: To prove that there exists a discretionary inflation level π^D such that π^D is optimal given π^e, and vice versa, we will use Brouwer's fixed point theorem. Since we are only interested in the universe of limited inflation, we state that $\pi^D \in [0, \overline{\pi}]$ where $\overline{\pi} > 0$ is an upper limit for the possible inflation levels. Let $\pi : [0, \overline{\pi}] \to [0, \overline{\pi}]$ be the function mapping private agents' expectations into the policymaker's inflation choice as defined by the discretionary inflation problem in Equation 7.

Let us now define the auxiliary function $\tilde{\pi}(\pi^D) := \pi(f\pi^D + (1 - f)\pi^a) = \pi(\pi^e)$. Since $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ maps a compact interval on $\mathbb{R}$ into itself, we only need to prove that it is continuous to use Brouwer's theorem for the existence of a fixed point.

First, by assumption, we know that the penalty function $\psi : [0, \overline{\pi}] \to \mathbb{R}_+$ mapping discretionary inflation into total factor productivity is continuous, assuming that the policymaker already chose to deviate from the target. Hence, the consumption choice will also be continuous. The same holds for government spending.

Second, the utility function $u : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ mapping government spending and private consumption into a utility scale is also continuous by assumption.

Combining the mapping of discretionary inflation $[0, \overline{\pi}]$ into consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ and the mapping of consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ into a utility scale $\mathbb{R}$, it is clear that the mapping of discretionary inflation $[0, \overline{\pi}]$ into a utility scale $\mathbb{R}$ will also be continuous. Finally, given that the argmax operator, mapping $[0, \overline{\pi}]$ into $[0, \overline{\pi}]$, maintains those properties, we have that $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ is continuous, which is what we sought to demonstrate.

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A Tables

Parameter	Value	Meaning	Calibration
β	.915	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π^a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Expansive gov.
ρ	.50	Pref. for consumpt.	Neutral value
ϵ	.004	Fixed cost	Brazilian 2002 crisis
κ	0.35	Welfare cost	Campos and Cysne (2018) estimation for 10% inflation cost

Table I: Parameters of the Baseline Model

Initial Debt Level	Debt Zone	Expected Inflation	Years to Exit the FFZ
60%	Credibility	3.5%	-
75%	Fiscal Fragility	4.6%	1 year
85%	Fiscal Fragility	4.9%	3 years
100%	Fiscal Dominance	12.8%	-

Table II: Expected Inflation Rates and the Time at Which the FFZ is Exited in the Calibrated Model for the Brazilian Case

Initial Debt Level	Optimal Inflation Target
65%	2.4%
75%	4.5%
85%	6.4%

Table III: Optimal Inflation Target for Initial Debt Levels in the Calibrated 2002 Brazilian Case

Year	Date When Set	Target	Bounds
2002	28/6/2000	3.50	2.0
2003	28/6/2001	3.25	2.0
	27/6/2002	4.00	2.5
	21/1/2003	8.50	
2004	27/6/2002	3.75	2.5
	21/1/2003	5.50	
	25/6/2003	5.50	2.5
2005	25/6/2003	4.50	2.5

Table IV: Brazil – Official Inflation Targets

	Debt/GDP	Revenue/GDP	CPI EOY	CPI target
Average	45.2	32.9	3.9	3.2
Min	13.4	16.4	1.5	1.5
Max	80.8	56.1	15.4	8.2

Table B.I: Data Description

	I	II	III	IV	V
Revenue	0.171^{**}	0.098	0.063	0.125^{*}	0.087
	(0.076)	(0.076)	(0.078)	(0.070)	(0.072)
Debt	0.069^{*}	0.074^{**}	0.073^{**}	0.062^{**}	0.058^{*}
	(0.035)	(0.034)	(0.034)	(0.031)	(0.032)
Debt * Revenue/100	-0.194^{*}	-0.168^{*}	-0.149	-0.163^{*}	-0.136
	(0.099)	(0.096)	(0.096)	(0.088)	(0.089)
Target	-0.403^{***}	-0.458^{***}	-0.441^{***}	-0.360^{***}	-0.342^{***}
	(0.063)	(0.062)	(0.062)	(0.059)	(0.058)
GDP Gap			0.363^{***}		0.342^{***}
			(0.102)		(0.095)
Reer YoY				-13.956^{***}	-13.645^{***}
				(1.648)	(1.653)
FE	Country	Country & Time	Country & Time	Country & Time	Country & Time
R^2	0.290	0.408	0.433	0.515	0.537
Num. obs.	382	382	374	372	364

Note: *p<0.1; **p<0.05; ***p<0.01

Table B.II: Results – Deviations from the Inflation Target

	I	II	III	IV	V
Revenue	0.145	0.108	0.084	0.115	0.082
	(0.091)	(0.105)	(0.109)	(0.110)	(0.113)
Debt	0.034	0.055	0.053	0.050	0.042
	(0.044)	(0.052)	(0.053)	(0.053)	(0.054)
Debt*Revenue/100	-0.114	-0.107	-0.088	-0.121	-0.085
	(0.125)	(0.149)	(0.151)	(0.151)	(0.154)
Target	-0.624^{**}	-1.242^{***}	-1.207^{***}	-0.990^{**}	-0.936^{**}
	(0.263)	(0.376)	(0.376)	(0.390)	(0.386)
GDP Gap			0.206		0.218
			(0.158)		(0.167)
Reer YoY				-10.493^{***}	-10.262^{***}
				(3.062)	(3.101)
Num. obs.	377	377	369	368	360
Log Likelihood	−178.526	−151.367	−149.281	−139.619	−137.954

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table B.III: Results – Probability of Overshooting the Target

Parameter	Value	Meaning	Calibration
β	.915	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π^a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Expansive gov.
ρ	.50	Pref. for consumpt.	Neutral value
σ	20%	Limit to TFP cost	Brazilian 2002 crisis
ϵ	.002	Fixed cost	Brazilian 2002 crisis
λ	1.77	Welfare cost	Campos and Cysne (2018) estimation for 10% inflation cost

Table C.I: Parameters of the Baseline Model

Initial Debt Level	Debt Zone	Expected Inflation	Years to Exit the FFZ
60%	Credibility	3.5%	-
75%	Fiscal Fragility	4.5%	1 year
85%	Fiscal Fragility	4.7%	4 years
100%	Fiscal Dominance	11.5%	-

Table C.II: Expected Inflation Rates and the Time at Which the FFZ is Exited in the Calibrated Model for the Brazilian Case

Initial Debt Level	Optimal Inflation Target
60%	2%
70%	4%
80%	5.5%

Table C.III: Optimal Inflation Target for Each Debt Level in the Calibrated 2002 Brazilian Case

B Figures

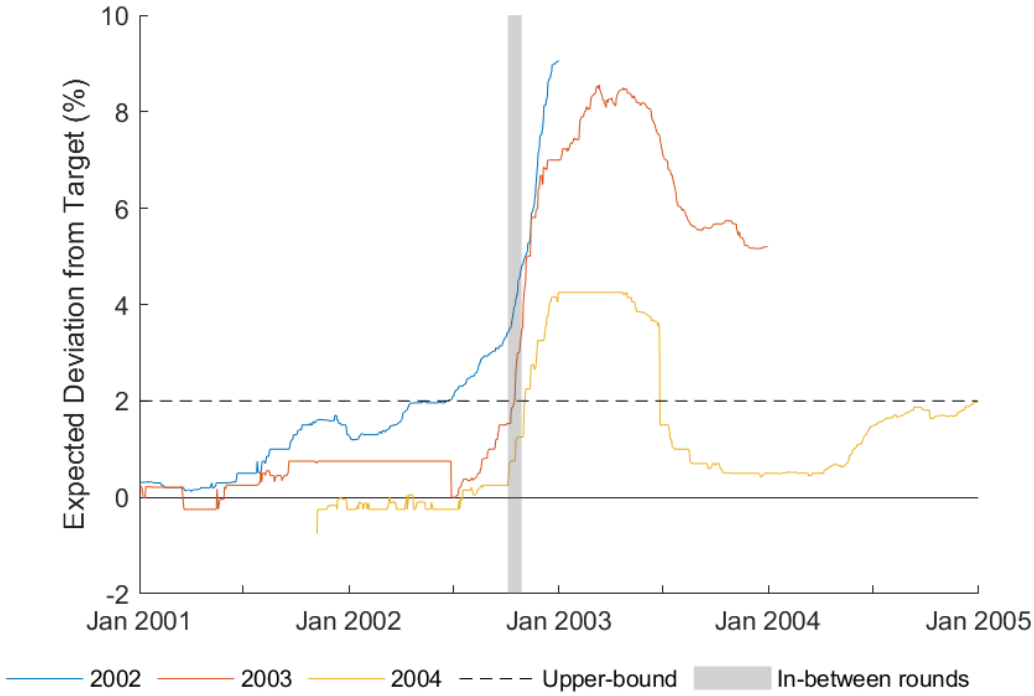

Figure I: Expectation Crisis in Brazil

This figure shows the inflation expectation crisis that happened in Brazil in 2002. On the y-axis, we plot the expected inflation for the end of the year minus the inflation target for that year. Expected inflation is the mean expected inflation by professional forecasters, collected by the Central Bank of Brazil and available at The Focus – Market Readout. On the x-axis, we plot the date when expected inflation was formed. Until October 2002, expected inflation was within the inflation target bands. However, between rounds of the Presidential election – shaded grey region – inflation expectations exceeded the target's upper bounds at all horizons relevant to the central bank (current year, 1-year ahead, and 2-years ahead).

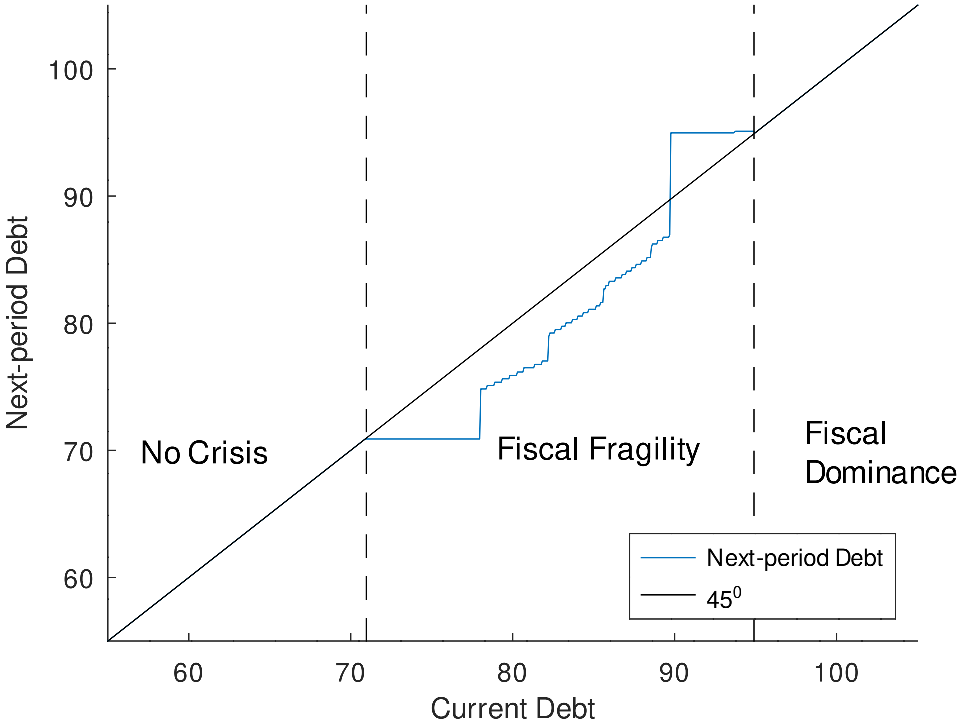

Figure II: Debt Policy Function

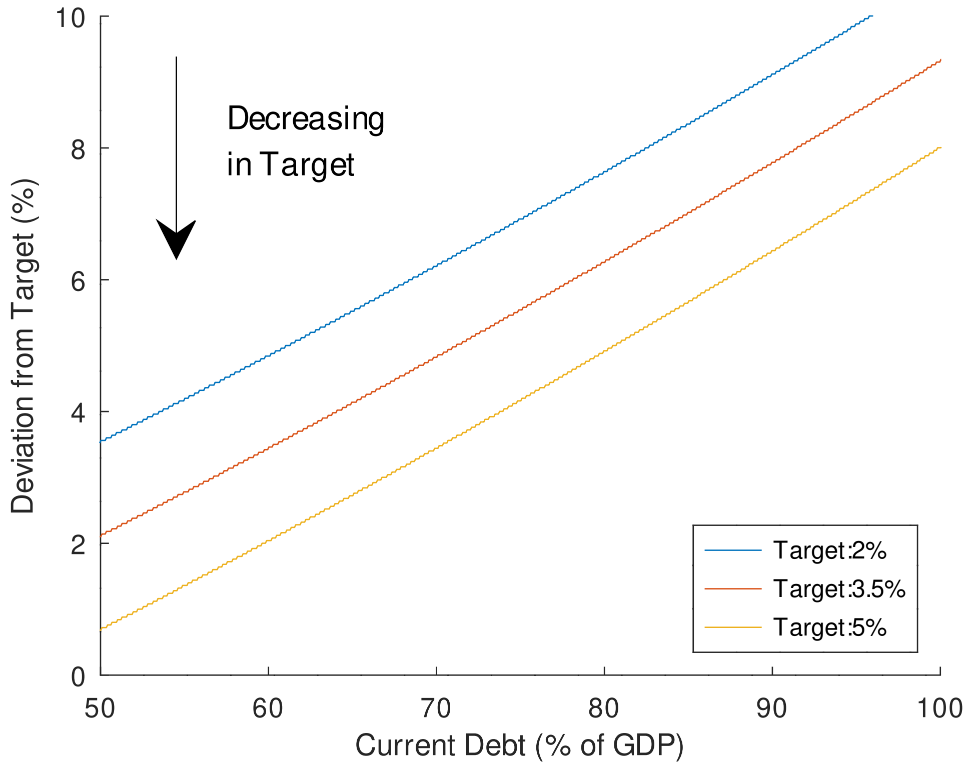

Figure III: Sensitivity: Deviations to Inflation Target

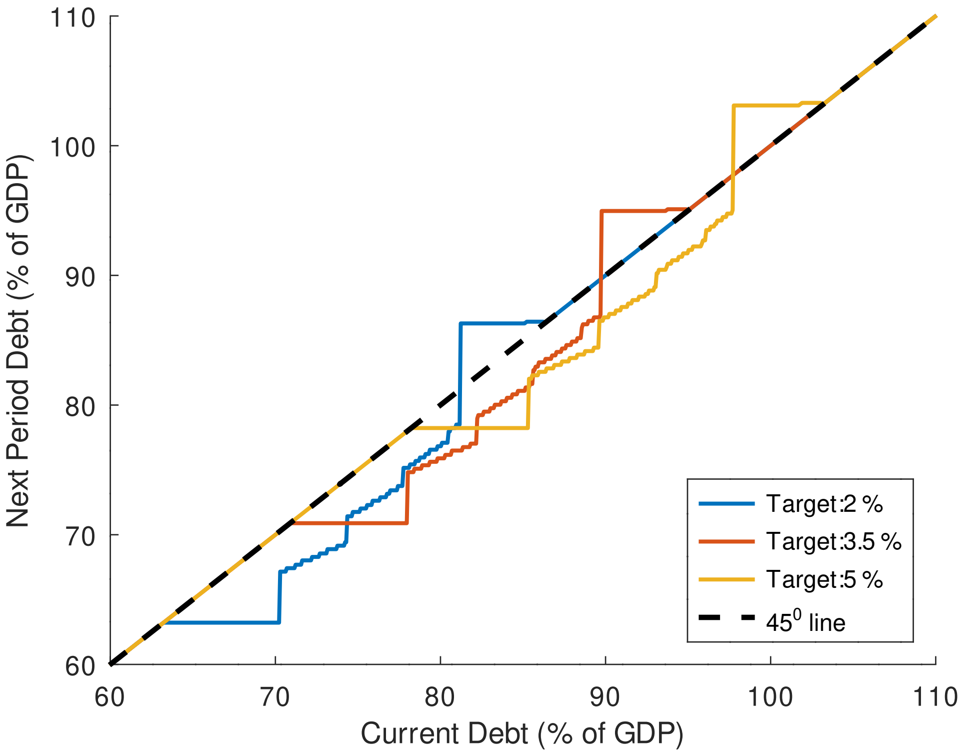

Figure IV: Sensitivity: Inflation Target and the Fiscal Fragility Zone

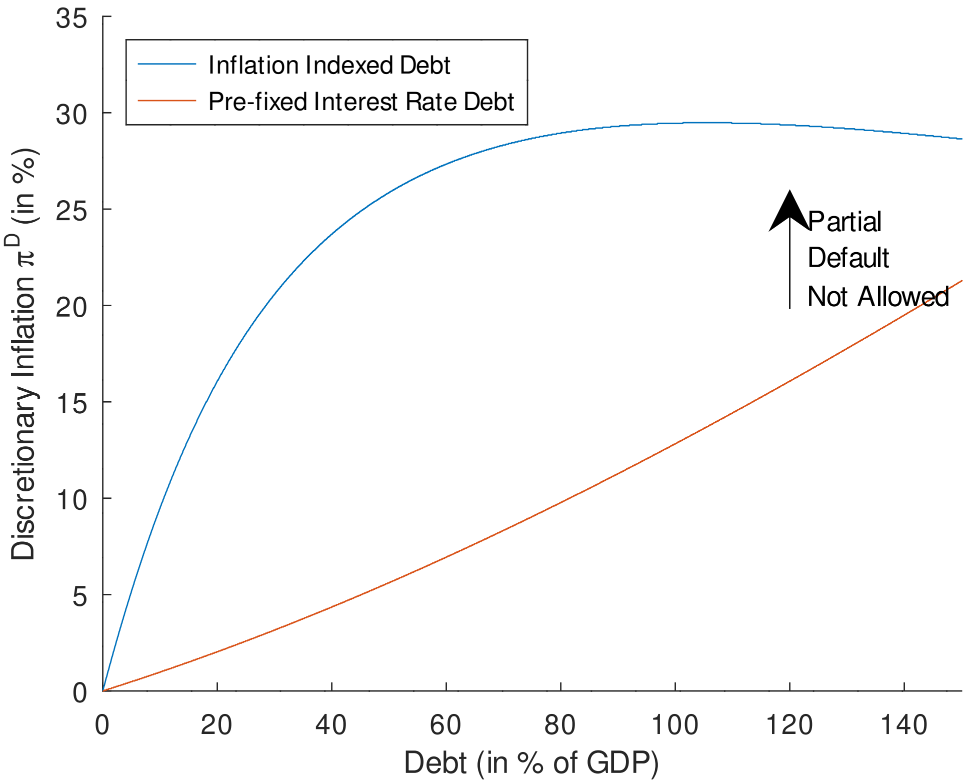

Figure V: Discretionary Inflation

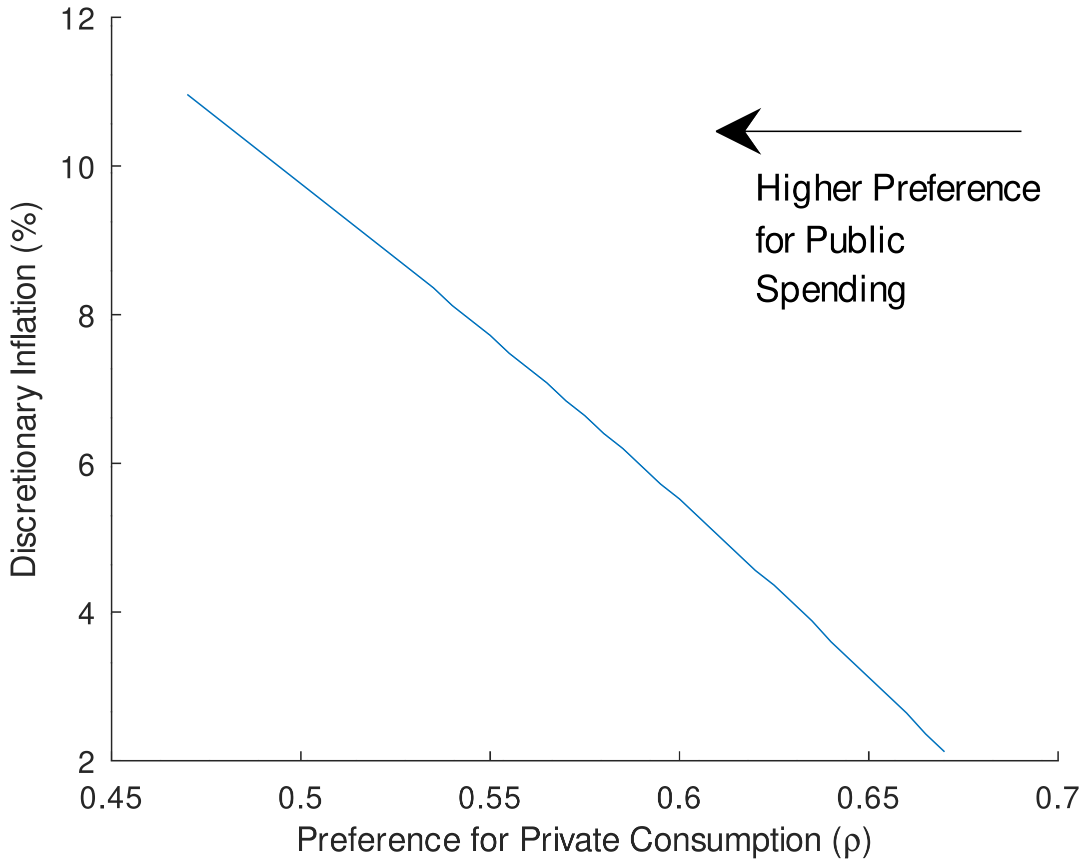

Figure VI: Sensitivity: Preference for Public Spending when $D = 80\%$

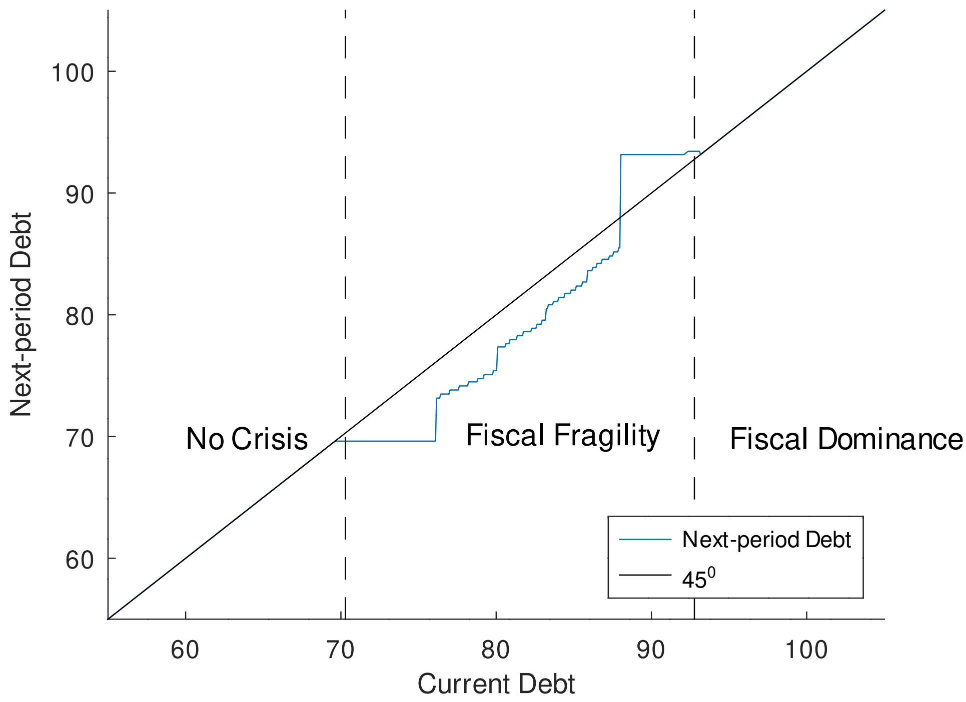

Figure C.I: Debt Policy Function

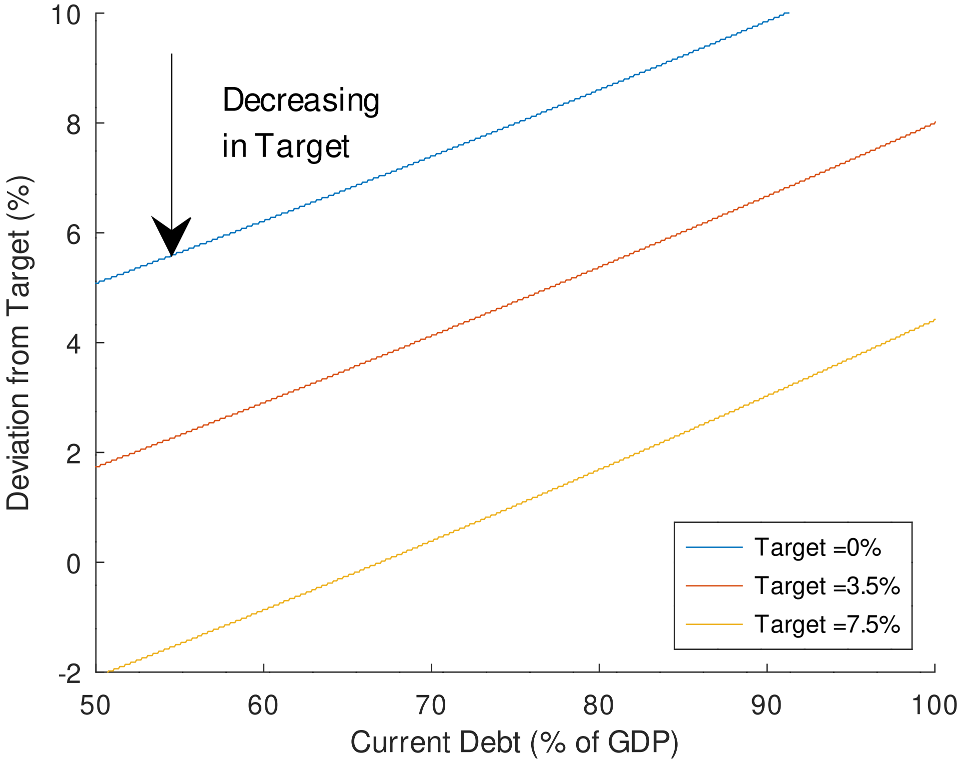

Figure C.II: Sensitivity: Deviations to Inflation Target

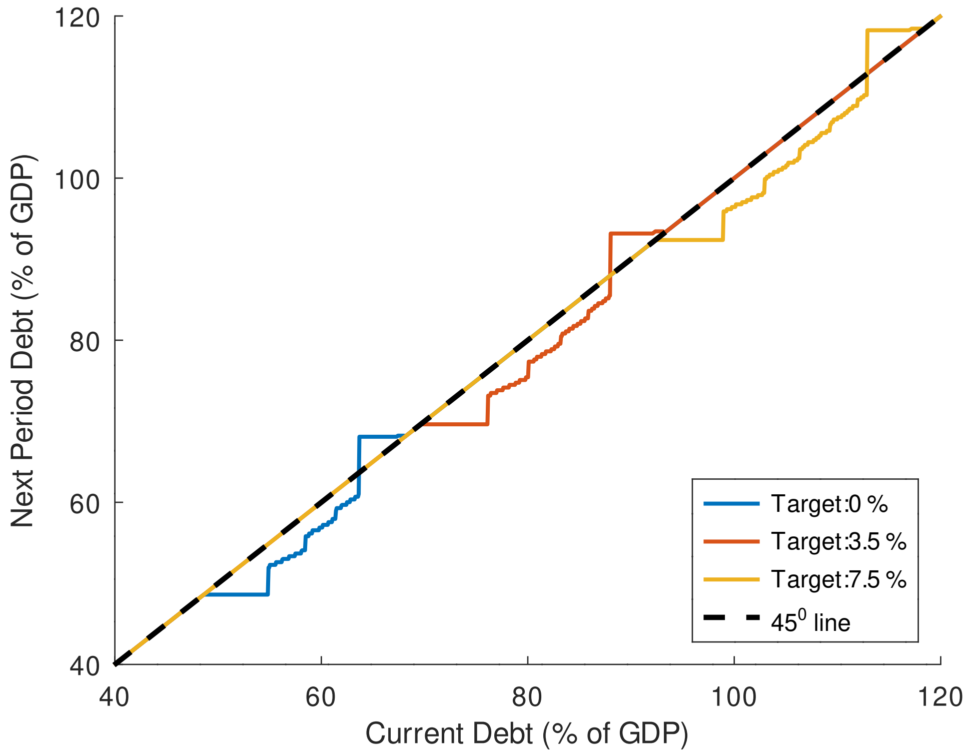

Figure C.III: Sensitivity: Inflation Target and the Fiscal Fragility Zone